# Actuarial Mathematics

Prof. Francesco Della Corte; Prof. Gian Paolo Clemente

Life insurance module: Prof. Francesco Della Corte

***COURSE AIMS AND INTENDED LEARNING OUTCOMES***

This course aims to develop mathematical methods to formalise life insurance contracts. Students will learn the calculation tools to price the products and evaluate the commitments of the insured and the insurer through actuarial value within a balanced market context.

*Knowledge and Understanding*

At the end of the course, students are expected to have acquired the basics of life expectancy representation and the corresponding needs to be insured.

*Ability to apply Knowledge and Understanding*

Students are expected be able to communicate the risk factors associated with survival and describe the characteristics of the typical products offered on the market.

They should also be able to distinguish the balance conditions between the insured and the insurer concerning the subsequent placement in the financial statements.

***COURSE CONTENT***

* *Fundamental biometric functions*: biometric functions - survival functions - mortality tables
* By using the statistical data and the corresponding life tables, suitable probabilities are assigned to life and death events.
* *Traditional life insurance contracts:* the main contracts concerning life and death for individual policyholders.
* The point of view of the insurer is presented to set benefits both in the demographic and financial setting.
* *Annuities:* Actuarial value of an annuity for a single individual - the annuities in pension products.
* *Premium calculation:* basic rules with loading conditions and expenses. The student will be able to state the equilibrium payment corresponding to the promised benefit, relative to the different products.
* *Risk and savings premiums and mathematical reserve formation in the Local GAAP context:* prospective, retrospective and recurring reserve - risk and savings premium. Having highlighted the different timing of the Insurance Company's commitment and the premiums payment, once the technical bases have been established, the student learns to evaluate the provision to reserve
* *Selected and projected tables:* construction of mortality tables both differentiated within a selection period, and which consider the trends in the death probabilities. Therefore, the student learns to build the aforementioned tables using deterministic models suitable for the projection of demographic scenarios.
* *Life insurance reserving:* the policy reserve for some insurance products under the various time approaches.
* *Linked policies:* indexed insurance - linked policies. The student reaches an overview of market consistent valuation and the need to know how to price, under the non-arbitrage assumption, elementary derivative contracts. With this purpose, the student will be able to price put and call options in a discretized context using the Cox-Ross-Rubinstein model.

***READING LIST***

*Course material:*

E. Pitacco, *Matematica e tecnica delle assicurazioni sulla durata di vita*, Trieste.

Additional material will be made available on the online Blackboard platform.

*Suggested Readings*

P. Mazzoleni, *Matematica attuariale - assicurazioni sulla vita,* EDUCatt, 2014

(on the actuarial mathematics topics proposed in the course)

E. Pitacco, et al. *Modelling longevity dynamics for pensions and annuity business*. Oxford University Press, 2009.

(On mortality forecasting models)

S. Shreve, *Stochastic calculus for finance I: the binomial asset pricing model*. Springer Science & Business Media, 2005.

(On the valuation of European options in the discrete time)

E. Giusti, *Analisi Matematica I*. Bollati Boringhieri, 2002.

(On the concepts of derivative and integral)

A.Tarelli, A.Sbuelz, *Quantitative Finance: Problems and Solutions*. Giappichelli, 2021.

(On non-arbitrage models)

***TEACHING METHOD***

Lectures and guided practical work

***ASSESSMENT METHOD AND CRITERIA***

The exam is based on both a written and an oral examination. The written examination is done on Excel. The compulsory oral exam can be taken only if the written exam has been passed. In case the exam has been failed, both written and oral examination must be taken again.

The students’ assessment consists on an interview on the entire course content. The exam is aimed at evaluating the students’ reasoning skills and analytical rigour on the topics covered in the course, as well as their command of the language and communication skills. The relevance of the students’ answers, their appropriate use of specific terminology, an argumentative and consistent structure of the speech, along with their ability to identify conceptual links and open issues will contribute to the assessment.

***NOTES AND PREREQUISITES***

Prerequisite for the students is a basic knowledge of General Mathematics, Quantitative Methods for Finance 1 and Excel.

Further information can be found on the lecturer's webpage at http://docenti.unicatt.it/web/searchByName.do?language=ENG or on the Faculty notice board.

Property-casualty module (property-casualty insurance): Prof. Gian Paolo Clemente

***COURSE AIMS AND INTENDED LEARNING OUTCOMES***

*Course aims*:

The course aims to provide students with a rigorous and systematic analysis of the most significant decision-making problems concerning damage insurance operations.

The course covers the following topics: the main forms of insurance coverage against damages, the determination of the fair premium, the need for a safety loading, expense loadings and determination of premium rates, a priori premium personalisation, a posteriori personalisation and bonus-malus systems, technical reserves, statistical-actuarial methods for the determination of claims reserves, reinsurance, solvency.

*Intended learning results*:

At the end of the course, students will be able to determine the risk and rate premiums for insurance coverage against damages. They will know the main deterministic methods for evaluating the claims reserve and will be able to distinguish the strengths and weaknesses of the methods analysed. Finally, students will know the main problems related to the exercise of the damage insurance necessary to ensure a correct technical and financial balance.

***COURSE CONTENT***

The course will cover the following topics:

* *The type of insurance coverage against damages and the main statistics of the Italian market.* The course will also illustrate the main types of insurance coverage, the main contractual clauses and the main insurance market trends.
* *Determination of the risk premium.* The method of calculation of the risk premium will be described using an empirical and a theoretical approach. The concepts of claims frequency and average cost will be introduced. The course will also describe the behaviour of the two factors in the main insurance branches.
* *Construction of the rate premium.* The course will introduce the concept of security loading and charges loading and present the main calculation methods for safety loading.
* *Personalisation of the premium.* The course will describe a priori and a posteriori premium personalisation techniques. Particular attention will be paid to rating in the field of Motor Civil Liability (Motor TPL).
* *Technical management indicators.* The course will describe the main indicators of technical management. Particular attention will be paid to the definition of the Loss Ratio, Combined Ratio and Run-Off of the claims reserve.
* *Direct compensation*. The course will illustrate the current insurance reimbursement procedure in the Motor TPL class, also known as CARD (DCA - Direct Compensation Agreement between Insurers). The main technical aspects of the agreement and the possible effects on the rating method will also be described.
* *Technical reserves.* The course will describe the main technical reserves included in the statutory financial statements of civil liability insurances.
* *The calculation of the premium reserve in the accounting balance sheet.* The course will describe the method for determining the premium reserve in the statutory financial statements. Particular attention will be paid to the calculation of the premium fractional reserve, the unexpired risk reserve and the additional reserves.
* *Statistical-actuarial methods for the evaluation of claims reserves*. The course will introduce the main statistical-actuarial methods for evaluating claims reserve. For this purpose, the triangular structure typically used in this context will be described. Finally, the course will describe the Paid Chain-Ladder and Fisher-Lange methodologies that can be used for the evaluation of the claims reserve.
* *Risk retention and reinsurance methods.* The course will introduce the main risk retention methodologies and describe the different reinsurance forms, by highlighting its main characteristics*.*
* *Solvency from its origins to the Solvency II project.* The course will describe the criteria for assessing the solvency of an insurance company and the evolution of the relevant legislation from its origins to the introduction of the Solvency II Directive.

***READING LIST***

*Course material:*

The main reference material is taken from:

N. Savelli-G.P. Clemente, *Lezioni di Matematica Attuariale delle Assicurazioni Danni*, Serie Materiale e Documenti, pag. 1-184, EduCatt, Milano, 2014.

Additional material (Excel file and market publications) will be made available on the online Blackboard platform.

*Possible further reading:*

L. Daboni, *Lezioni di tecnica attuariale delle assicurazioni contro i danni*, Lint, Trieste, 1993.

P. Gigante-L. Picech-L. Sigalotti, *La tariffazione nei rami danni con modelli lineari generalizzati*, EUT (Cap. 1, Cap. 2), 2010.

S.A. Klugman-H.H. Panjer-G.E. Willmot, *Loss Models: From Data to Decisions*, 4th Edition, Wiley, 2012.

A. Olivieri-E. Pitacco, *Introduction to Insurance Mathematics. Technical and Financial Features of Risk Transfers*, Springer, 2010.

E. Pitacco, *Elementi di matematica delle assicurazioni*, Lint, Trieste, 2002.

G. Werner-C. Modlin, *Basic Ratemaking*, Casualty Actuarial Society, 2016.

M.V. Wuthrich, *Non-Life Insurance: Mathematics & Statistics*, 2019 Available at SSRN: https://ssrn.com/abstract=2319328 or http://dx.doi.org/10.2139/ssrn.2319328

***TEACHING METHOD***

Classroom lectures in Italian. Use of sample files in Excel.

***ASSESSMENT METHOD AND CRITERIA***

The exam is based on both a written and an oral examination. The written examination is based on multiple choice questions. The compulsory oral exam can be taken only if the written exam has been passed. In case the exam has been failed, both written and oral examination must be taken again.

The students’ assessment covers the entire course content. The exam is aimed at evaluating the students’ reasoning skills and analytical rigour on the topics covered in the course, as well as their command of the language and communication skills. The relevance of the students’ answers, their appropriate use of specific terminology, an argumentative and consistent structure of the speech, along with their ability to identify conceptual links and open issues will contribute to the assessment.

***NOTES AND PREREQUISITES***

*Prerequisites*

Before attending the course students will have to know the basic notions of General Mathematics and the concepts of discrete and continuous random variables, the concepts of probability function, density and distribution functions, the concepts of mean, variance and asymmetry and the methods of moments, and the main discrete and continuous probability distributions.

Further information can be found on the lecturer's webpage at http://docenti.unicatt.it/web/searchByName.do?language=ENG or on the Faculty notice board.