# Game Theory

## Prof. Andrea Calogero

***COURSE AIMS AND INTENDED LEARNING OUTCOMES***

The aim of the course is to show how situations of interactive choices can be studied from a mathematical point of view so as to improve the results obtained by individuals and the community. To do this, students will be taught the principal ideas and fundamental tools of mathematical game theory, including the presentation of a few applicative examples.

At the end of the course, students will be able to:

describe everyday situations from the game theory point of view;

build mathematical models that describe situations of interaction, discuss their main aspects, and recognise the limits and potential of the models proposed;

prove the main results of game theory;

analyse and solve examples of cooperative and non-cooperative games.

***COURSE CONTENT***

1. INTRODUCTION TO GAME THEORY

* Preferences, utility functions and their relationship.
* Utility functions and linear utilities (by von Neumann and Morgenstern).

2. STRATEGIC GAMES

* Some examples and models of strategic games.
* The Nash equilibrium: correspondences and related properties, Nash Theorem.
* Two-players zero-sum games: lower value, upper value and value of the game.
* Mixed strategies in finite games. Bimatrix game and matrix game. Von Neumann Theorem.
* Refinement of Nash equilibrium in finite game: perfect equilibrium, dominated strategies.

3. EXTENSIVE GAMES

* Some examples and models of extensive games.
* Definition of extensive game. Game with perfect information and perfect recall.
* Pure strategies, mixed strategies and behaviour strategies. Kuhn Theorem.
* The Nash equilibrium in extended games with different strategies. The equilibrium existence theorem.
* Subgame perfect equilibrium.

4. COOPERATIVE GAMES:

- Definition of the cooperative game

- Bargaining problems.

- Transferable utility games. Solution concepts: the core. The Shapley value.

***READING LIST***

J. González-Diaz, I. García-Jurado, M.G. Fiestras-Janeiro, *An introductory course on mathematical game theory,* American Mathematical Society

R. Lucchetti, *A primer in game theory,* Esculapio, 2011.

M. Maschler, E. Solan, S. Zamir, *Game theory,* Cambridge University Press, 2013.

A. Calogero, R. Pini, *Esercizi di teoria dei giochi,* available on the website of the course.

***TEACHING METHOD***

Classroom lectures or through e-learning tools (depending on the course of the health emergency).

***ASSESSMENT METHOD AND CRITERIA***

A written exam.

The written exam comprises a number of exercises and open-ended questions with demonstrations. The written exam assesses the student's knowledge of the topics covered during the course, their ability to solve the exercises, and their ability to demonstrate the principal theorems and apply the main results in different contexts.

The oral exam is used to comment on the results of the written exam with the students. During the oral exam, students will be assessed on their ability to correct any mistakes made, contextualise the exercises within the theory, and use the mathematical tools presented during the course.

The exams may be carried out electronically depending on the course of the health emergency.

***NOTES AND PREREQUISITES***

Students must possess a basic knowledge of mathematical analysis.

*Further information can be found on the lecturer's webpage at http://docenti.unicatt.it/web/searchByName.do?language=ENG or on the Faculty notice board.*