# Geometric Structures

## Prof. Silvia Pagani

COURSE AIMS AND INTENDED LEARNING OUTCOMES

The course aims to introduce geometric structures over finite fields and their applications. On the one hand, the affine and projective spaces studied in the courses of the Bachelor degree will be taken up again and viewed from a new perspective, in which knowing how to *count* is fundamental; on the other hand, we will introduce some areas based on finite geometries, such as the Coding Theory and (part of) Graph Theory.

At the end of the course, students will be able to:

- understand the concepts and entities introduced in the theory, correctly express their definitions and properties, and know their mutual links;

- state and rigorously prove theorems, and be able to find their consequences;

- use counting techniques to identify the cardinality of the structures presented and to deduce significant geometric properties;

- have a unifying vision of affine and projective geometry over any field, capturing common aspects and differences between finite and infinite fields;

- discuss the positioning of a code among the optimal ones;

- identify the main characteristics of a graph and be able to state the main results and conjectures related to it.

COURSE CONTENT

First part: Geometries over finite fields

Review of finite fields. Projective plane. Construction of the Fano plane. Projective and affine spaces. Cardinality of a projective space. The number of k-dimensional projective subspaces in an n-dimensional space. Subgeometries, Baer subgeometries. Conics and quadrics. Polarity. Hermitian varieties. Non-Desarguesian projective planes: examples of order 9. Blocking set. Arcs, caps and applications.

Second part: Coding Theory

Transmission model. Hamming distance, t-correcting codes, weight of a codeword. Linear codes, generating matrix. Dual code, parity check matrix. Syndrome-decoding algorithm. Parameters of a linear code and known bounds. Optimal codes.

Third part: Graph theory

The origin of graph theory: the Seven Bridges of Königsberg. Introductory notions. Classes of graphs. Eulerian and Hamiltonian graphs. Trees. Linear graph. Colouring of vertices and edges. Open problems on graphs.

READING LIST

First part:

JWP HIRSCHFELD, *Projective geometries over finite fields*, 2nd edition, Oxford University Press, Oxford (1998).

JWP HIRSCHFELD - JA THAS, *General Galois Geometries*, Oxford Math. Monogr., Oxford University Press, Oxford (1991).

Second part:

L. GIUZZI, *Codici correttori. Una introduzione*, Springer (2006).

FJ MACWILLIAMS - NJ SLOANE, *The theory of error-correcting codes*, North-Holland, Amsterdam (1977).

Third part:

F. HARARY, *Graph theory*, Addison-Wesley, Reading, MA (1969).

DB WEST, *Introduction to graph theory*, 2nd edition, Prentice-Hall (2001).

TEACHING METHOD

Classroom lectures and practical exercises.

ASSESSMENT METHOD AND CRITERIA

An oral exam aimed at ascertaining the student's knowledge of the concepts, results and procedures illustrated during the course, through a presentation and discussion of some of the points covered in the programme and the links between them.

The assessment of the exam will take into account the correctness of the procedures illustrated, their logical and methodological rigour, and the efficacy and accuracy of the presentation; a student's ability to assimilate the concepts, personally rework them and summarise facts will be particularly valued.

NOTES AND PREREQUISITES

The basic knowledge required to follow this course is that covered in the Geometry I, Further Study of Geometry and Further Study of Algebra courses, the latter for the part relating to finite fields, which will be reviewed at the beginning of the course. Maximum attention should always be paid to the language and meaning of the symbols that will be gradually introduced, and to the logical rigour of the discussion and the importance of the links between the entities introduced.

*Further information can be found on the lecturer's webpage at http://docenti.unicatt.it/web/searchByName.do?language=ENG or on the Faculty notice board.*