# Statistical Mechanics and Complex Systems

Prof. Fausto Borgonovi

***COURSE AIMS AND INTENDED LEARNING OUTCOMES***

The course is divided into two parts: in the first part the student will learn the basic notions as well as the tools necessary for the study of classical and quantum equilibrium statistical mechanics. In the second part, these tools will be used in the study and analysis of some modern applications of statistical physics, such as phase transitions, stochastic processes and models of classical and quantum chaos. At the end of the course, the student should be able to frame a generic problem of statistical physics from the physical modelling point of view, to determine the relevant set of equations that determine its behaviour and to identify the appropriate exact or approximate solution schemes, analytical or numerical as well.

***PROGRAM (part I)***

***1 -***  **The Statistical Foundations of Thermodynamics:** Microscopic and Macroscopic states.The classical ideal gas. Mixing entropy and the Gibbs paradox. Phase space. Liouville theorem. Microcanonical, Canonical and Gran Canonical Ensembles. Partition function. Ensemble equivalence and fluctuations. The harmonic oscillators gas. Equipartition theorem. Density and energy fluctuations in the gran canonical ensemble.

***2 –* Quantum Statistical Mechanics**: Density Matrix. Statistics of the various ensembles. Systems composed of undistinguishable particles. The ideal gas in the quantum ensembles. Statistics of occupation numbers. The density matrix and the partition function in a system of free particles.

***3 –* Quantum Gas** : Ideal Bose gas. Bose-Einstein condensation. Thermodynamics of blackbody radiation. Ideal Fermi gas. Pauli paramagnetism. Landau levels. Aharonov-Bohm effect and flux quantization. Landau diamagnetism.

***PROGRAM (part II)***

***4 –* Phase transitions** : Classification. I and II order phase transitions. The Van der Wall gas. Ferromagnetic phase transition (phenomenology). Heisenberg and Ising models. 1-D Ising model (exact solution). Mean field theory. Landau mean field theory. Correlation functions. Fluctuations-Dissipation theorem. Critical exponents. Scaling hypothesis and scale invariance.

***5 -* Stochastic Processes** : Langevin equation and fluctuation-dissipation theorem. Time dependent correlation functions. Einstein relation. Brownian motion in a thermal oscillator gas. The Wiener-Khinchin theorem. The Liouville equation. The Fokker-Planck equation. Master equations. Optical absorption coefficient.

**6 - Classical and Quantum Chaos :** Integrable and non-integrable systems. Classical perturbation theory. Hamiltonian systems as canonical mappings. KAM stability. Poincarè-Birkhoff theorem. Linearized motion. Standard map. Ergodicity. Lyapunov exponents. Kolmogorov-Sinai entropy. diffusion in action space. Randomness. Quantum Chaos. Level Statistics. Random Matrix Theory. Dynamical localization.

***BIBLIOGRAPHY***

1. K. Huang, *Statistical Mechanics,* J. Wiley & sons, New York (1987).

2. R.K.Pathria, *Statistical Mechanics,* Elsevier Science (1996)

3. B. Cowan, *Topics in Statistical Mechanics,*  Imperial College Press, London (2005).

4. R.Zwanzig, *Non equilibrium Statistical Mechanics,* Oxford Univ. Press (2001).

5. A.J. Lichtenberg – M.A.Lieberman , *Regular and Stochastic Motion ,* Springer-Verlag New York (1983).

***TEACHING METHOD***

Frontal lessons for both the first part (30 hours) and the second part (30 hours). During the lessons, exercises or completions of demonstrations are given to be delivered as a "written exam" a few days before the oral proof.

***ASSESSMENT METHOD AND CRITERIA***

The exam consists of an oral test divided into three parts. In the first part, the exercises carried out at home by the student will be discussed. Correct resolution of the test will contribute up to 20% of the final mark. In the second part, the candidate will present a short report about an argument freely chosen (even if the choice should beinherent to the issuesaddressed during the second part of the course). The topics covered during the second part of the course must be considered only as a starting point for the choice of presentation and superpositions with the the lessons should be avoided. The presentation can be organized with the aid of IT tools. This test will compete for a score up to 30% of the final mark. The oral exam then proceeds with a series of questions aimed at ascertaining the assimilation of the main topics covered, the mastery of the subject and the ability to connect topics and concepts presented within the course. The evaluation of the oral exam takes into account the correctness of the illustrated procedure, their rigorous logic and methodology, the effectiveness and correctness of the presentation, enhancing the assimilation and the personal reworking. This part contributes to the global evaluation up to a maximum of 50%.

***NOTES AND PREREQUISITES***

A good knowledge of Thermodynamics, Classical and Quantum Mechanics are essential requirements.

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