# Dynamical Systems

## Prof. Alessandro Musesti

***COURSE AIMS AND INTENDED LEARNING OUTCOMES***

The course aims at introducing the student to the qualitative analysis of discrete and continuous dynamical systems, providing some knowledge about mathematical models based on systems of ODE and on iterated maps, modeling phenomena taken from mechanics, biology and economics. At the end of the course the student will manage the concepts of equilibrium and stability and will be able to qualitatively study some mathematical models. He/She will also manage the main modeling techniques of biological and economic phenomena and will learn the main features of chaotic models.

***COURSE CONTENT***

Matrix exponential. Ordinary differential equations. Properties of solutions. Equilibrium and stability. Stability of linear systems. The two-dimensional case. Isoclines. Stability by linearization. Lyapunov function. Equilibrium instability. Chetaev Instability Theorem.

Models of population dynamics: Malthusian growth model, continuous logistic model, Lotka-Volterra equations. Commensalism, mutualism, competition. Epidemiological models: SIS and SIR models. Supply and demand economic models. Macroeconomic models.

Semigroups and processes. Attractors. Limit cycles. Poincaré-Bendixon Theorem. Bifurcation of equilibrium. Lorenz system. Deterministic chaos.

Discrete dynamical systems. Spectral radius. Stability of equilibrium for discrete systems. Periodic orbits. Sharkovsky’s Theorem. One-dimensional maps. Logistic map. Tent map.

***READING LIST***

L. Perko, *Differential Equations and Dynamical Systems,* Springer.

G. Gaeta, *Modelli Matematici in Biologia*, Springer.

Additional lecture notes will be provided during the course.

***TEACHING METHOD***

Classroom lectures, sometimes complemented with computer simulations.

***ASSESSMENT METHOD AND CRITERIA***

There will be both a written and an oral examination. The written examination lasts 60 minutes and consists of 4 or 5 exercises: it will test the expertise of the students in studying continuous and discrete dynamical systems and the stability of equilibria. The oral examination will evaluate the knowledge of the student about the theoretical topics of the course and the analysis of specific models. The relevance of the answers, the appropriate use of specific terminology and the coherent structuring of the exposition will contribute to the assessment.

There is a unique final mark, assessing the written test for 40% and the oral test for the remaining.

***NOTES AND PREREQUISITES***

Some basic notions in Calculus, Mechanics and Linear Algebra are required. The main useful concepts will be recalled during the course.

*Further information can be found on the lecturer's webpage at http://docenti.unicatt.it/web/searchByName.do?language=ENG or on the Faculty notice board.*