**Stochastic Processes**

Prof. Luca Tamanini

***COURSE AIMS AND INTENDED LEARNING OUTCOMES***

The course aims to provide the students an introduction to stochastic differential equations and to their applications.

At the end of the lectures, any student will be able to model simple physical and financial problems with stochastic differential equations, to solve them, to understand their utility in applications and to understand the mathematical theory which lays below them.

***COURSE CONTENT***

Reminders of elementary probability: probability spaces, random variables, stochastic processes, expected value, variance, probability distribution functions, stochastic independence, Chebychev inequality, Borel-Cantelli lemma, characteristic functions, central limit theorem, conditional expectation, martingales.

Brownian motion: definition and basic properties, rigorous construction of a one-dimensional Brownian motion and some ideas about the n-dimensional Brownian motion, non-differentiability of sample paths. Basic notions on absolutely continuous functions and functions of bounded variation.

Stochastic integrals: construction of Ito integral and its main properties, stochastic differentials, product rule and chain rule, the vectorial case.

Stochastic differential equations (SDE): Definition and examples, existence and uniqueness of solutions, elementary properties of solutions, linear SDEs and explicit formulas for the solutions. Basic notions on the theory of distributions, the white noise as distributional derivative of the Brownian motion.

Some applications: stopping time, link between SDE and PDE, Feynman-Kac formula, optimal stopping.

Basic concepts about stocks, bonds, derivatives, options in financial markets, derivation and analysis of the Black and Scholes model related to the fair pricing of an option. Some properties of the heat equations.

***READING LIST***

L. C. Evans, *An Introductio to Stochastic Differential Equations,* American Mathematical Society 2013.

R.N. Mantegna, H.E. Stanley*, An introduction to Econophysics: Correlations and Complexity in Finance*, Cambridge University Press, 2007.

L. Arnold, *Stochastic Differential Equations: Theory and Applications*, Wiley, 1973.

L. Tamanini, *Lecture notes.*

***TEACHING METHOD***

Lectures at the blackboard.

***ASSESSMENT METHOD AND CRITERIA***

Oral exam. The interview is designed to ascertain the extent to which the students have assimilated the concepts, results and procedures illustrated during the course, through explaining and discussing some of the points of the course program, not excluding references to prerequisites or relationships between the parts of the program.

The grading of the interview will take into account the accuracy of the concepts illustrated, their logical and methodological rigor, and the effectiveness and accuracy in explanation, with a value assigned to the assimilation of concepts and the reworking thereof by the student.

***NOTES AND PREREQUISITES***

Students are expected to be familiar with the knowledges of the Mathematical Analysis courses from the Bachelor. Knowledge about *L^p* spaces is preferrable, though not mandatory. For more information the students can contact the lecturer with an email.

*Further information can be found on the lecturer's webpage at http://docenti.unicatt.it/web/searchByName.do?language=ENG or on the Faculty notice board.*