# Mathematical Logic

## Prof. Antonino Ventura

***COURSE AIMS AND INTENDED LEARNING OUTCOMES***

The course aims to provide students with the notions and tools for building a formal language, and with a general understanding of its use in propositional and predicate logic.

At the end of the course, students will be able to describe the specific characters of formal languages, use extensional connectives and quantifiers, know the syntax and semantics of propositional and predicate logic, understand the rules of metatheoretical calculus, demonstrate semantic metatheorems, specify the concept, constitutive elements and general properties of formal theories, identify their metatheoretical properties, clearly grasping the aim of finitary formalisation and, in particular, that related to the axiomatic component of theories.

Furthermore, students will be able to use and apply the demonstrative procedures of natural deduction calculus formulated for sequences in various ways, distinguishing the rules that can only be classically derived from the rules derived in intuitionistic or minimal predicate calculus.

Finally, students will be able to grasp the importance of logic for computer science and its computational developments and will have the ability to integrate and communicate the skills acquired both in interdisciplinary contexts and in the mathematical-foundational sector.

***COURSE CONTENT***

1. *Formal language of deduction*

- Nature, object and functions of logic

- Concept of proposition. Simple and complex propositions. Structure of a proposition. Propositional connectives. Quantifiers

- Determining the truth value of complex propositions. Defining certain connectives by means of others

- Formal language in building a calculus

2. *Propositional logic*

- Syntax of classic propositional logic

- Elements of intuitionist and minimal calculus

- Truth of a formula in an interpretation. Notions of satisfiability, logical consequence and validity

- Semantics of classical propositional logic

- Correctness theorem, completeness theorem, and semantic finiteness (or compactness) theorem for classical propositional calculus

3. *Logic of predicates*

- Syntax of predicate logic

- Basic semantic notions

- Semantics of first order predicate logic

- Correctness theorem and completeness theorem for predicate calculus

- Consequences of semantic completeness

- Finiteness as a precondition of formality. The problem of axiomatising theories. Non-categorisation of elementary arithmetic (Skolem theorem) and Peano second order arithmetic categorisation (Dedekind theorem)

***READING LIST***

S. Galvan, *Logica*,La Scuola, Brescia 2012.

E. J. Lemmon, *Elementi di logica*, Laterza, Rome-Bari 2009.

E. Mendelson, *Introduzione alla logica matematica*, Bollati Boringhieri, Turin 1972.

G. Lolli, *Introduzione alla logica formale*, Il Mulino, Bologna 1991.

Further reading references will be communicated during the course.

***TEACHING METHOD***

Lectures.

***ASSESSMENT METHOD AND CRITERIA***

An oral exam.

Assessment will be based on the following: *a*) verification of students' knowledge by way of three or more questions of equal weight, aimed at demonstrating the extent and depth of preparation; *b*) verification of students' specific skills and abilities, evidenced by their correct and effective use of formal language.

Relevance of answers, ability to identify conceptual links and appropriate use of specific technical terminology will be assessed in general. With regard to the content of the answers and the presentation, the following criteria will apply: *a*) assessment of students' ability to process the language of predicate calculus and to use it in metatheoretical arguments; *b*) verification of students' knowledge of demonstrative procedures and ability to present them in a rigorous and straightforward fashion.

***NOTES AND PREREQUISITES***

For successful course attendanceand to pass the related exam, students are requiredto have knowledge of set theory and, in general, of the main contents of basic mathematics.

Further information can be found on the lecturer's webpage at http://docenti.unicatt.it/web/searchByName.do?language=ENG or on the Faculty notice board.