# Elements of higher geometry (9 cfu)

## prof. Mauro Spera

***COURSE AIMS AND INTENDED LEARNING OUTCOMES***

The course provides an introduction to topological and differentiable manifolds together with tensor analysis. Subsequently, the elements of Riemannian and symplectic geometry and of the theory of Lie groups will be discussed. The course will be quite concrete and will be based on examples also emerging in other parts of mathematics. The student will eventually acquire command of important concepts and techniques also needed in different theoretical domains in applications.

***COURSE CONTENT***

1.Prologue  
Multilinear algebra. Topological and differentiable manifolds. Tensor analysis. Frobenius' theorem. Lie groups. Lie group actions on manifolds.  
Quotient manifolds. Homogeneous spaces.  
  
2. Riemannian geometry  
Affine connections. Riemannian manifolds. Levi Civita's connection. First variation of the energy functional. Geodesics. Exponential map. Curvature tensors (Riemann, sectional, Ricci, scalar). The Hopf-Rinow theorem. Second variation of the energy functional. Jacobi fields. Conjugate points. Examples and applications.  
  
3. Symplectic geometry  
Symplectic manifolds. Hamiltonian mechanics. Momentum maps. Examples and applications.

***READING LIST***

M. Spera``*Differential geometry and topology* ", available online on Blackboard.

Extra information can be found in:

V.I. Arnold, Méthodes Mathématiques de la Méchanique Classique, MIR, Moscou, 1976.  
W. Boothby*,  An introduction to differentiable manifolds and Riemannian geometry*,  
 Academic Press, New York, 1975.  
R. Bott, L.T. Tu, *Differential forms in algebraic topology* Springer, New York, 1982.  
S.S Chern,  *Complex manifolds without potential theory*, Springer-Verlag, Berlin, 1979.  
S.S Chern,  H. Chen,  K.S. Lam, *Lectures on differential geometry*, World Scientific, Singapore, 2000.  
M. Do Carmo,  *Riemannian Geometry*, Birkhaeuser, Boston, 1992.  
M. Do Carmo ,  *Differential Forms and Applications*, Springer, Berlin, 1994.  
B. Dubrovin, A, Fomenko, S.Novikov, *Géométrie Contemporaine*, (3 vol.) MIR, Moscou, 1982  
A.T. Fomenko, T.L. Kunii, *, Topological Modeling for Visualization*. – Springer-Verlag, 1997.  
J. Gallier*, Geometric Methods and Applications for Computer Science and  
Engineering*, Springer, Berlin, 2000.  
S. Gallot,  D. Hulin,  J. Lafontaine , *Riemannian Geometry*, Springer, 1987.  
G. Gentili,  F. Podestà, E. Vesentini*,  Lezioni di geometria differenziale*. Bollati-Boringhieri, Torino, 1995.  
S. Goldberg, *Curvature and Homology*, Dover, New York, 1962. \par  
J.M. Lee,   *Introduction to Topological Manifolds*, Springer-Verlag, Berlin, Heidelberg, New York, 2000.  
J.M. Lee,  *Introduction to Smooth manifolds,*  Springer-Verlag, Berlin, Heidelberg, New York, 2003.  
J.M. Lee*,   Riemannian geometry: an introduction to curvature*,  Springer-Verlag, Berlin, Heidelberg, New York, 2003.  
E. Sernesi,  *Geometria 2,*  Bollati Boringhieri, Torino, 1994.  
I.M. Singer, J.A. Thorpe  *Lezioni di topologia elementare e di geometria*, Boringhieri, Torino, 1980.

***TEACHING METHOD***

Traditional classroom lectures.

***ASSESSMENT METHOD AND CRITERIA***

At the end of the course there will be an oral test aiming at verifying the students’ learning outcomes. Assessment will take place through an oral exam aiming at verifying the student’s level of assimilation of the concepts and theorems through exposition and discussion of some of the points of the syllabus, with possible connections to pre-requisite knowledge. The final evaluation will assess the candidates’ explanatory efficacy, clearness and accuracy, together with assimilation of the concepts and their own personal critical elaboration.

***NOTES AND PREREQUISITES***

Prerequisites involve standard content of a Bachelor Programme in Mathematics. Regular attendance is strongly encouraged.

***NOTES AND PREREQUISITES***

Prof. Spera will meet students in his office during lesson days and by appointment.

*Further information can be found on the lecturer's webpage at http://docenti.unicatt.it/web/searchByName.do?language=ENG or on the Faculty notice board.*

# Elements of higher geometry (6 cfu)

## prof. Mauro Spera

***COURSE AIMS AND INTENDED LEARNING OUTCOMES***

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***COURSE CONTENT***

1.Prologue  
Multilinear algebra. Topological and differentiable manifolds. Tensor analysis. Frobenius' theorem.   
  
2. Riemannian geometry  
Affine connections. Riemannian manifolds. Levi Civita's connection. First variation of the energy functional. Geodesics. Exponential map. Curvature tensors (Riemann, sectional, Ricci, scalar). The Hopf-Rinow theorem. Second variation of the energy functional. Jacobi fields. Conjugate points. Examples and applications.  
  
3. Symplectic geometry  
Symplectic manifolds. Hamiltonian mechanics. Examples and applications.

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M. Do Carmo ,  *Differential Forms and Applications*, Springer, Berlin, 1994.  
B. Dubrovin, A, Fomenko, S.Novikov, *Géométrie Contemporaine*, (3 vol.) MIR, Moscou, 1982  
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S. Gallot,  D. Hulin,  J. Lafontaine , *Riemannian Geometry*, Springer, 1987.  
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