# Elements of Higher Algebra (9 ECTS)

## Dr. Marco Antonio Pellegrini

# Elements of Higher Algebra (6 ECTS)

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***COURSE AIMS AND INTENDED LEARNING OUTCOMES***

The aim of this course is to provide the basic notions of Theory of Modules on a ring, with particular attention to the finitely generated modules on a principal ideals domain. The second part of the course will focus on Homological Algebra, where we introduce concepts as tensor product, esact sequence, category and functor.

At the end of the course, the student will be able to appreciate the relevance of the theory of modules, with its numerous applications. He will also be able to classify finitely generated abelian groups and to study the endomorphisms of vector spaces through their canonical forms (rational and of Jordan). He will also master fundamental notions of Homological Algebra, which will allow him to begin to study of this important part of Mathematics.

***COURSE CONTENT***

- Review of Algebra and Linear Algebra: the course begins with a brief review of the concepts seen regarding groups and rings, vector spaces, matrices and determinants on commutative ring, study of a linear application.

- Modules on a ring: once we have seen the definitions and the first properties, we consider concepts as submodules, homomorphisms, quotient modules and direct sums.

- Some important classes of modules: we consider in particular, finitely generated modules, torsion modules and free modules.

- Free modules on PID: focusing on this particular class of modules, we study the submodules of a free module on a principal ideals domain, considering the equivalence of matrices and the related invariant factors.

- Finitely generated PID modules: this part is the heart of this course. We prove the fundamental structure theorem for finitely generated modules on a principal ideals domain. The primary decomposition and the invariance theorem is also studied.

- Applications of the structure theorem: the previous theorem is applied to study finitely generated abelian groups and the canonical forms of matrices.

- Tensor product of modules: we define the tensor products of modules using biadditive functions. Then, we study some properties of this algebraic construction, considering in particular the case of commutative rings and vector spaces.

- Esact sequences and esact functors: we study esact sequences of modules over a ring, focusing on some classical results. Then, we introduce the concepts of category and functor. In particular, we study functors between the category of modules over a ring and the category of abelian groups.

***READING LIST***

The main reference is the handouts provided by the teacher on the platform Blackboard. For a deeper understanding of the topics, we recommend:

- M. Curzio, P. Longobardi and M. Maj, *Lezioni di Algebra,* Liguori Editore, 1994.

- B. Hartley and T.O. Hawkes, *Rings,* Modules and Linear Algebra, Chapman & Hall, London-New York, 1980

- N. Jacobson, *Basic Algebra I: Second Edition,* Dover Books on Mathematics, 2009.

- N. Jacobson, *Basic Algebra II: Second Edition,* Dover Books on Mathematics, 2009.

***TEACHING METHOD***

The course consists 60 hours of theoretical lessons on the blackboard and exercises on the topics seen in class. Students can find numerous exercises and the text of previous written exams in the lecture notes provided by the teacher.

***ASSESSMENT METHOD AND CRITERIA***

The exam consists of a written test and an oral test. The student must first pass a written test, of the duration of 2 hours, consisting of 4 exercises (each of them with the value of 7-8 points) covering the topics seen in the first part of the course. The test is considered passed if the student gets at least 18 points out of 30. The result of this test will be published, in accord with the laws on privacy, on Blackboard.

Once this test has been passed, the student during one of the exam sessions (chosen by the student), must pass an oral test, which has a duration of 45 minutes and in which the theoretical preparation is tested and the student's exhibition capacity. During this oral exam, the student will be asked to describe and test some theorems seen throughout the course, including the second part concerning Homological Algebra.

The final result is unique and is based on the results of both the written and oral proves.

***NOTES AND PREREQUISITES***

It is assumed that the student already has basic notions of Linear Algebra and Algebra. Usually these notions are acquired during the first year of the Mathematics degree course. The student is invited to a quick review before the course starts. A certain interest in abstract algebra is also assumed.

Further information can be found on the lecturer's webpage at http://docenti.unicatt.it/web/searchByName.do?language=ENG or on the Faculty notice board.

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***COURSE AIMS AND INTENDED LEARNING OUTCOMES***

The aim of this course is to provide the basic notions of Theory of Modules on a ring, with particular attention to the finitely generated modules on a principal ideals domain. The second part of the course will focus on the theory of finite group representations, where the notions seen in the first part of the course will be applied to study finite groups.

At the end of the course, the student will be able to appreciate the relevance of the theory of modules, with its numerous applications. He will also be able to classify finitely generated abelian groups and to study the endomorphisms of vector spaces through their canonical forms (rational and of Jordan). He will also master fundamental notions of Representation Theory, which will allow him to begin, if he wishes, the study of groups using their characters.

***COURSE CONTENT***

- Review of Algebra and Linear Algebra: the course begins with a brief review of the concepts seen regarding groups and rings, vector spaces, matrices and determinants on commutative ring, study of a linear application.

- Modules on a ring: once we have seen the definitions and the first properties, we consider concepts as submodules, homomorphisms, quotient modules and direct sums.

- Some important classes of modules: we consider in particular, finitely generated modules, torsion modules and free modules.

- Free modules on PID: focusing on this particular class of modules, we study the submodules of a free module on a principal ideals domain, considering the equivalence of matrices and the related invariant factors.

- Finitely generated PID modules: this part is the heart of this course. We prove the fundamental structure theorem for finitely generated modules on a principal ideals domain. The primary decomposition and the invariance theorem is also studied.

- Applications of the structure theorem: the previous theorem is applied to study finitely generated abelian groups and the canonical forms of matrices.

***READING LIST***

The main reference is the handouts provided by the teacher on his own web page and on the platform Blackboard. For a deeper understanding of the topics, we recommend

* M. Curzio - P. Longobardi and M. Maj, *Lezioni di Algebra,* Liguori Editore, 1994.
* B. Hartley and T.O. Hawkes, *Rings,* Modules and Linear Algebra, Chapman & Hall, London-New York, 1980
* N. Jacobson, *Basic Algebra I: Second Edition,* Dover Books on Mathematics, 2009.

***TEACHING METHOD***

The course consists of 40 hours of theoretical lessons on the blackboard and exercises on the topics seen in class Students can find numerous exercises and the text of previous written exams in the lecture notes provided by the teacher.

***ASSESSMENT METHOD AND CRITERIA***

The exam consists of a written test and an oral test. The student must first pass a written test, of the duration of 2 hours, consisting of 4 exercises (each of them with the value of 7-8 points) covering the topics seen in the first part of the course. The test is considered passed if the student gets at least 18 points out of 30. The result of this test will be published, in accord with the laws on privacy, on Blackboard.

Once this test has been passed, the student during one of the exam sessions (chosen by the student), must pass an oral test, which has a duration of 45 minutes and in which the theoretical preparation is tested and the student's exhibition capacity. During this oral exam, the student will be asked to describe and test some theorems seen throughout the course.

The final result is unique and is based on the results of both the written and oral proves.

***NOTES AND PREREQUISITES***

It is assumed that the student already has basic notions of Linear Algebra and Algebra. Usually these notions are acquired during the first year of the Mathematics degree course. The student is invited to a quick review before the course starts.

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