# Geometry I

## Prof. Silvia Maria Carla Pagani

COURSE AIMS AND INTENDED LEARNING OUTCOMES

The course aims to provide an initial introduction to Geometry as a formal language for describing reality, starting from the theory of vector spaces.

It also aims to provide the fundamental notions of Linear Algebra, in order to introduce students to the language of vector spaces as a powerful and elegant formal tool for the most varied mathematical and non-mathematical applications, in particular for the theory of systems and for an analytical introduction of metric, affine and projective geometry.

At the end of the course, students will be able to:

- understand the concepts and entities introduced in the theory, correctly express their definitions and properties, and know their mutual links;

- rigorously enunciate the theorems, know exactly where to use them and the respective implications, and provide proofs for some of them;

- autonomously perform the exercises presented in the supplementary teaching and build examples and counterexamples;

- apply the knowledge learned in theory to solving the different types of problems related to each topic.

COURSE CONTENT

Vector spaces

Geometric vectors and operations on them. Groups and fields: definitions and examples. The notion of a vector space: definition, examples and first properties; linear dependence and independence, bases, dimension, subspaces and operations between them, Grassmann formula.

Homomorphisms between vector spaces: kernel, image and related theorems; isomorphism between vector spaces of finite dimension *n* on a given field K; spaces of homomorphisms, linear forms and dual space.

Matrices

Operations between matrices; determinant, Laplace and Binet theorems; invertibility of matrices and their rank; matrix representations of homomorphisms and basis changes for vector spaces of finite dimension, similarities between matrices.

Linear systems

Linear systems and scalar representations of homomorphisms between vector spaces, Rouché-Capelli and Cramer theorems, principles of system equivalence and elementary operations on matrices, Gaussian elimination method and reduction to scale of linear systems and matrices.

Parametric and Cartesian equations of vector subspaces.

Endomorphisms of a vector space

Eigenvectors, eigenvalues and eigenspaces, characteristic polynomial and criteria for diagonalisation of endomorphisms and square matrices.

Metric vector spaces

Bilinear forms: matrix representation (in finite dimension), basis changes and congruence between matrices. Scalar products: associated quadratic forms, orthogonality, isotropic vectors, orthogonal bases and their existence, canonical forms of complex and real quadratic forms (or symmetric matrices) (Sylvester's theorem).

Euclidean scalar products: norm, angles, orthogonal projections of vectors, orthonormal bases, Gram-Schmidt orthogonalisation theorem; vector product; orthogonal matrices, unit operators (isometries).

Affine, Euclidean and projective geometry

Affine spaces: definition, translations, subspaces, parallelism, geometric properties of affine spaces.

Coordinatization of an affine space of finite dimension, parametric and Cartesian equations of affine subspaces, equations of translations and affinities; ~~analytical~~ analytic geometry of affine spaces, with particular regard to the plane and three-dimensional space, pencils and stars of lines and planes.

Euclidean spaces: distance between two points, angles, orthogonality; Euclidean geometry in the plane and in space: orthogonality and distances between lines, between planes, between lines and planes, circumferences and spheres, isometries; some geometric loci.

Projective plane; homogeneous coordinates of points and equations of lines in the real and complex projective plane.

Real plane algebraic curves

General notions on real algebraic curves in the real and complex projective plane: order, simple and singular points, tangent lines, reducibility.

Conics: projective classification, conic pencils, polarity; affine classification, centre, diameters, asymptotes; metric classification, axes, foci and focal properties, metric canonical equations.

READING LIST

M. Abate, *Geometria,* McGraw-Hill Libri Italia srl, Milan, 1996.

TM Apostol, *Calcolo,* Vol.2, Geometria. Bollati Boringhieri, Turin, 1986.

P. DULIO, W. Pacco, *Algebra Lineare e Geometria Analitica,* Esculapio, Bologna, 2015.

E. Sernesi, *Geometria 1,* Bollati Boringhieri, Turin, 1989.

E. Zizioli, *Algebra ed Elementi di Geometria,* *Temi d’esame con svolgimento commentato degli esercizi*, Cartolibreria Snoopy, Brescia, 2010.

Lecture notes written by the course lecturer will also be provided.

TEACHING METHOD

Classroom lectures and practical exercises.

ASSESSMENT METHOD AND CRITERIA

A written and an oral exam.

The written exam will consist of some exercises in which students must show that they have acquired the skills in Linear Algebra and Analytical Geometry and know how to apply them to specific situations similar to those illustrated in the hours of supplementary teaching.

The evaluation of the written exam will take into account the correctness of the results and the procedures used to obtain them, as well as the quality of their presentation.

The oral exam is aimed at ascertaining the student's knowledge of the concepts, results and procedures illustrated during the course, through a presentation and discussion of some of the points covered in the programme and the links between them.

The assessment of the oral exam will take into account the student's accuracy of the procedures illustrated, their logical and methodological rigour, and the efficacy and accuracy of their presentation; a student's ability to assimilate the concepts and personally rework them will be particularly valued.

NOTES AND PREREQUISITES

A general upper secondary school level of knowledge is the basic requirement for taking this course. Maximum attention should always be paid to the language and meaning of the symbols that will be gradually introduced, as well as to the logical rigour in dealing with them.

Further information can be found on the lecturer's webpage at http://docenti.unicatt.it/web/searchByName.do?language=ENG or on the Faculty notice board.