. - **Geometry II**

Prof. Mauro Spera

***COURSE AIMS AND INTENDED LEARNING OUTCOMES***

The course introduces and elaborates the fundamental ideas of the differential geometry of curves and surfaces, in a rigorous yet concrete and example-based manner, so as to further develop the students' geometric intuition, abstraction and analytical computing ability, also in view of applications to parallel and successive courses.

*1. Differential geometry of plane and spatial curves and surfaces*

Regular parametric curves. Arc length. Plane curves: arc length in polar coordinates.

Plane curves: (signed) curvature, radius of curvature, osculating circle and its characterization as the limit of circles tangent to the curve in a point and passing through another point of the curve, as the latter point tends to the given one. General formula for the curvature, complex and “mixed” formalisms. Reconstruction of a plane curve from its curvature up to a rigid motion (fundamental theorem of plane curves), explicit formula. Examples: lines, conics and other classical curves (cycloid, tractrix, clothoid etc.). Evolutes and evolvents.

The evolute of a tractrix is a catenary, the evolute of a cycloid is a cycloid.

Spatial curves: curvature, biregularity, principal trihedron, torsion, Frénet-Sérret formulae. Fundamental theorem (curvature and torsion determine a curve up to a rigid motion), with idea of proof. Local study of a (biregular) curve via the Frénet trihedron. Dini's theory. Osculating sphere and de Saint Venant's theorem. Examples: twisted cubic, elices, Viviani's curve.

*2. Differential geometry of surfaces*

Review of vector calculus.

Regular parametric surfaces. First fundamental form (metric). Mercators' map. Stereographic projection (with its property of mapping circles into circles). Metric on surfaces of revolution; Beltrami's pseudosphere.

The Gauss map and the shape operator. The second fundamental form and its geometric interpretations (Meusnier's theorem; shifting from the tangent plane) principal curvatures, asymptotic lines, lines of curvatures and Rodrigues' theorem. Euler's theorem. Dupin indicatrix. Gaussian and mean curvature and their explicit formulae. Principal curvatures and its geometrical significance (meridian curvature and inverse of the grandnormal). Curvature of the pseudosphere. Examples (helicoid, catenoid...).

Weingarten's formulae. The “Theorema Egregium” and the Codazzi-Mainardi equations (general scheme). Curvature formulae. Covariant derivative and its geometric interpretation (Levi-Civita). Christoffel symbols. Proof of the Theorema Egregium. Fundamental theorem of surface theory (hint). Parallel transport and its geometric significance. Parallel transport on the sphere.

[Prologue: review of analytical mechanics. Stationary action principle and Lagrange equations, cyclic coordinates and ensuing conserved quantities (first integrals)]. Geodesics and their intrinsic and extrinsic properties: autoparallel curves, critical paths for energy and length functionals (with respect to the arc length), curves vith vanishing geodesic curvature (definition of the latter and geometrical significance, with proof). Examples of geodesics (euclidean plane,

sphere, hyperbolic plane, surfaces of revolution (Clairaut's theorem).

Gauss formula for geodesic triangles. Application to non-euclidean geometries: sphere, projective plane (elliptic plane), hyperbolic plane. The Gauss-Bonnet theorem.

Remarks on the exponential mapping, normal and polar coordinates, geodesic circles, Gauss lemma and intrinsic characterization of curvature (Minding's theorem and Bertrand-Puiseaux formulae.

Examples, exercises and complements, computational techniques: quadrics, developables, ruled surfaces, minimal surfaces and their variational characterization (helicoid, catenoid...).

***READING LIST***

M.Spera, *Elementi di geometria differenziale,* lecture notes available on Blackboard.

**Further references**

M.Abate - F.Tovena, *Curve e superfici,* Springer, Milano, 2006.

M.Do Carmo, *Differential Geometry of Curves and Surfaces,* Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1976.

A.Gray - E.Abbena - S.Salamon, *Modern Differential Geometry of Curves and Surfaces with Mathematica,* CRC Press, Boca Raton, 2006.

D.Hilbert - S.Cohn-Vossen, *Geometria intuitiva,* Boringhieri, Torino, 1972.

M.Lipschutz, *Geometria differenziale Schaum,* Etas Libri, 1984.

A.Pressley, *Elementary Differential Geometry,* UTM Springer, New York, 2000.

E.Sernesi, *Geometria 2 Bollati Boringhieri,* Torino, 1994.

***TEACHING METHOD***

Traditional blackboard lectures and class exercises.

***ASSESSMENT METHOD AND CRITERIA***

The final assessment will take place through an oral exam aiming at verifying the student’s level of assimilation of the concepts and theorems through the students’ exposition and discussion of some of the points of the syllabus, with possible connections to pre-requisite knowledge. The final evaluation will assess the candidates’ explanatory efficacy, clearness and accuracy, together with assimilation of the concepts and their own personal critical elaboration.

***NOTES AND PREREQUISITES***

Prerequisites involve standard content of the first year of a Bachelor Programme in Mathematics. Regular attendance is strongly encouraged.

***Office hours***

Prof. Spera will meet students in his office during lesson days and by appointment.

*Further information can be found on the lecturer's webpage at http://docenti.unicatt.it/web/searchByName.do?language=ENG or on the Faculty notice board.*