# Complements of Geometry

## Prof. Silvia Pianta

***COURSE AIMS AND INTENDED LEARNING OUTCOMES***

To introduce students to the basic notions of general topology and first notions of algebraic topology.

At the end of the course, students should be able to:

- understand the concepts and objects introduced in the theory, correctly express their definitions and properties, and know their mutual links;

- rigorously enunciate the theorems, know exactly where to use them and the respective implications, and provide proofs for some of them;

- independently perform the exercises suggested during the course, as well as build examples and counterexamples.

***COURSE CONTENT***

General topology

• Topological spaces: definitions and examples; interior, boundary, closure, accumulation points and isolated points; closed sets and dense subsets. Bases, metrizable spaces, and topologically equivalent metrics. Continuous functions and homeomorphisms.

• Subspaces, product topologies and quotient spaces: definitions, properties, examples. Topological embeddings, product maps, projections, quotient maps, identifications.

• Homeomorphism classes of spaces

• Topological properties: separation axioms (T1 and T2), compact, locally compact and relatively compact spaces, compattification, connected and path-connected spaces. Applications

Algebraic topology

• Homotopy: homotopy of continuous maps; homotopy equivalence: contractible spaces; retractions, retracts, deformation retractions and deformation retracts.

• Equivalence of paths and of loops with fixed base point, path product and homotopy invariance, constant and inverse path.

• Fundamental group: functorial properties, simply connected spaces.

• Calculation of some fundamental groups: paths and homotopy lifting and the fundamental group of the circle; Brouwer fixed point theorem; the fundamental group of product spaces: torus and cylinder; Seifert–van Kampen theorem and the fundamental group of the n-sphere for n>1; free groups and wedge sum of topological spaces.

***READING LIST***

C. Kosniowski., *Introduzione alla topologia algebrica,* Zanichelli, Bologna, 1988.

E. Sernesi, *Geometria 2,* Bollati Boringhieri, Turin, 2001.

A. Hatcher, *Algebraic topology,* <https://pi.math.cornell.edu/~hatcher/AT/AT.pdf> copyright by A. Hatcher, 2001.

Course lecture notes will be provided by the lecturer.

***TEACHING METHOD***

Lectures.

***ASSESSMENT METHOD AND CRITERIA***

At the end of the course there will be an oral test.

The oral test aims to verify the student’s level of assimilation of the concepts, results and procedures illustrated throughout the course. Students will be thus asked to expose and discuss some of the points of the syllabus, the connections between the several parts of the syllabus.

The assessment of the oral test will take into consideration the following aspects: the accuracy of the illustrated procedures, their logical and methodological rigour, the explanatory efficacy and accuracy. The final evaluation will also reward the candidate’s assimilation of the concepts and their own personal elaboration.

***NOTES AND PREREQUISITES***

The basic knowledge required to follow this course is that contained in the Geometry and Analysis courses of the first year of Mathematics. Maximum attention should always be paid to the language and meaning of the symbols that will be gradually introduced, as well as to the logical rigour in dealing with them.

*Further information can be found on the lecturer's webpage at http://docenti.unicatt.it/web/searchByName.do?language=ENG or on the Faculty notice board.*