# Functional Analysis

## Prof. Marco Squassina

***COURSE AIMS AND INTENDED LEARNING OUTCOMES***

To teach the basics of Sobolev spaces and elliptic equations. At the end of the course, students will know the rudiments of the existence and regularity of solutions of linear and non-linear elliptic problems.

***COURSE CONTENT***

**Part 1.** *Harmonic functions*: fundamental solution; average value formulae; principle of the maximum; derivative estimates; Liouville's theorem; analyticity; Harnack's inequality.

**Part 2.** *Sobolev spaces:* definitions and examples; some elementary properties; approximation with regular functions; extensions and trace; Sobolev's inequalities; Poincaré's inequality; compact immersions.

**Part 3.** *Linear elliptic equations*: weak formulation; existence via the Fredholm alternative; weak maximum principle; Hopf's lemma; strong maximum principle; H2 regularity.

**Part 4.** *Nonlinear problems:* elements of Calculus of Variations: first and second variations; weak lower semicontinuity; existence and uniqueness of minimums. Some classic examples: the brachistochrone; the catenary; the isoperimetric problem. Elements of critical point theory.

***READING LIST***

L. Evans, *Partial differential equations,* American Mathematical Society, 19, 1998.

H. Brezis, *Analisi funzionale,* *Teoria e applicazioni*, Liguori, Naples, 1986.

D. Gilbarg - NS Trudinger, *Elliptic partial differential equations of second order,* Grundlheren der athematischen Wissenschaften, 224, Springer-Verlag, Berlin-New York, 1977.

***TEACHING METHOD***

Lectures.

***ASSESSMENT METHOD AND CRITERIA***

An oral exam.

The oral exam is aimed at ascertaining how well the student has assimilated the concepts, results and procedures illustrated in class, through the presentation and discussion of a number of points covered in the programme.

The assessment of the oral exam will take into account the student's accuracy of the procedures illustrated, their logical and methodological rigour, and the efficacy and accuracy of their presentation; a student's ability to assimilate the concepts and personally rework them will be particularly valued.

The criteria used for the student's assessment and their resultant mark are based on the ability to summarise and analyse the notions imparted.

***NOTES AND PREREQUISITES***

The teaching is theoretical in nature and only a few exercises on the theorems demonstrated in class will be carried out. The prerequisites are a knowledge of basic analyses and the Higher Education Institution course.

*Further information can be found on the lecturer's webpage at http://docenti.unicatt.it/web/searchByName.do?language=ENG or on the Faculty notice board.*