# Elements of Numerical Analysis (9 CFU)

## Prof. Maurizio Paolini

# Elements of Numerical Analysis (6 CFU)

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***COURSE AIMS AND INTENDED LEARNING OUTCOMES***

To teach students the basis they need to solve limit problems numerically (discretization), with special reference to the finite element method for elliptic partial differential equations. Related aspects include: solving linear systems with modern iterative methods and function approximation techniques.

At the end of the course, students will be able to deal with the numerical resolution of simple mathematical models that lead to second or fourth order elliptical partial derivative problems (typically problems of minimum potential/heat/elastic energy for even complex two- or three-dimensional domains) using the finite element technique and currently available software tools, or by tackling the problem through the independent development of an appropriate calculus code.

***COURSE CONTENT***

Limit problems in several dimensions: Galerkin method and finite elements, interpolation error and energy-norm error estimates.

Elliptic equations (Poisson's equation): L2 error estimate.

Conditioning of the stiffness matrix.

Computational problems: generation of the grid, assembly of the matrices.

Adaptive techniques for partial differential equations (outline).

Notes on the case of parabolic and hyperbolic equations.

Linear systems with sparse matrices: gradient and conjugate gradient method for positive definite symmetrical matrices. Preconditioners for linear systems.

Computation of eigenvalues and eigenvectors of matrices: power method, similarity transformations, QR method.

***READING LIST***

V. Comincioli, *Analisi Numerica,* *Metodi Modelli Applicazioni,* McGraw Hill, Libri Italia, Milan, 1990.

A. Quarteroni - A. Valli, *Numerical approximation of partial,* differential equations, Springer, 1994.

C. Johnson, *Numerical solution of partial differential equations by the finite element method,* Cambridge university press, Cambridge, 1990.

G.H. Golub - C.F. Van Loan, *Matrix Computations,* The Johns Hopkins University Press,

Baltimore and London, 1993.

***TEACHING METHOD***

Lectures.

***ASSESSMENT METHOD AND CRITERIA***

An oral exam. The exam aims to ascertain students' assimilation of course concepts through a discussion on some of the its topics, focussing particularly on students’ ability to identify aspects of problems to be solved that are particularly relevant from a numerical point of view.

Assessment will take into account accuracy of students’ presentation, logical and methodological rigour and presentation efficacy, and personal reworking of the subject matter.

***NOTES AND PREREQUISITES***

Students should possess the basics for identifying the problems that may be numerically addressed with the techniques learned during the course, typically problems corresponding to mathematical models defined on a domain extended across various areas (materials science, phase transitions, form optimisation, image reconstruction, etc.) in which various types of equilibrium configurations are sought. They must possess the basic notions of Numerical Analysis and be familiar with the notions of Mathematical Analysis in relation to multiple variable functions.

Interested students may contact the lecturer and the tutor by email for more information.

*Further information can be found on the lecturer's webpage at http://docenti.unicatt.it/web/searchByName.do?language=ENG or on the Faculty notice board.*

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