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Ferdinando Colombo e Alessandra Mainini

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The control of politicians in modern democracies: discipline, selection and rent–shrinking

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Abstract

The public choice approach to politics sees a politician as a rational, self–interested agent who is bound to act within a given institutional setting, which is determined by either democratic decisions or spontaneous order.

We consider a two–period model where politicians differ in both their motivation and their competence. At the end of the first period, the citizens observe both the policy implemented and the incumbent politician’s competence. If they are not satisfied, they can take a series of actions that (partly) tie the politician’s hands in the future (first type of control of politicians) or simply do not reelect the incumbent politician and elect a new one (second type of control of politicians).

We show that the optimal control of politicians depends in a non–trivial way on the interplay of disciplining, selection and rent–shrinking effects. We also show that the society’s ability to use both types of controls may actually reduce social welfare.

Keywords: control of politicians, electoral accountability, politician’s motivation, anti–government demonstrations

JEL Classification: D720, D820

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1 Introduction

The public choice approach to politics sees a politician as a rational, self-interested agent who is bound to act within a given institutional setting, which is determined by either democratic decisions or spontaneous order. The rules that specify the length of the political mandate or who are the voters are examples of the first type of institutions, while articles by the free press or anti-government demonstrations are examples of the second type.

We study whether and how different institutions incentivate the politician to pursue the general interest in a simple two-period game. All the citizens are assumed to share the same view of social welfare, so the society as a whole can be considered as one of the players. The other player is a (female) elected politician, who has to choose whether to maximize social welfare or to extract a rent.

We initially consider the very simple case where politicians only differ in their motivation: a congruent politician is interested in social welfare, while a non-congruent politician only cares about rents. We firstly analyze the case where the incumbent politician always stays in office for two periods. If at the end of the first period the citizens are not satisfied with the policy implemented, they can (spontaneously) react in many democratic ways (first type of control of politicians). The hostile attitude of the society towards the incumbent politician somehow ties her hands, thus making it more difficult for her both to maximize social welfare and to extract rents. More institutional interpretations of the assumed ability of the citizens to (partly) tie the politician’s hands could, however, also be proposed. For example, one could imagine that at the end of the first period the citizens were asked to vote whether the politician should be constrained to a balanced-budget in the next period. Such a constraint would be able to avoid that a non-congruent politician extracts huge rents, but it might also not enable a congruent politician to maximize social welfare when this called for running a budget deficit. We show that in the unique perfect Bayesian equilibrium of the game, either the non-congruent politician maximizes social welfare in the first period in order to avoid a negative attitude of the citizens towards her in the following period, when she will be able to extract the full rent, or she extracts the full rent in the first period, but will then be able to extract only a partial rent in the following period. In the first case, the society’s ability to limit the politician’s power has a fully disciplining effect on the non-congruent politician, while in the second case it has a rent-shrinking effect. We then analyze the case where, at the end of the first period, the society chooses whether to reelect the incumbent politician (second type of control of politicians). We show that in the unique perfect Bayesian equilibrium of the game, the non-congruent politician maximizes social welfare in the first period in order to be reelected in the second period. The society’s ability not to reelect the incumbent politician has therefore a fully disciplining effect on the non-congruent politician. Finally, we compare the two types of controls of politicians. In our model, the disciplining effect is socially more important than the rent-shrinking effect; so, from
a social welfare point of view, the society’s ability not to reelect the incumbent politician dominates its ability to limit the politician’s power.

We then assume that politicians also differ in their competence. Competence is an intrinsic feature of a politician, which is observed by the society at the end of the first period and that, together with the policy implemented, determines whether the incumbent politician will be reelected. The society is now able to replace a low-ability politician with a new politician, who has on average a higher competence. This is the selection effect of reelection. We show that the existence of factors like competence, which are not under the control of the politicians but that affect their probability of being reelected, reduces the politicians’ incentive to maximize social welfare, and that the more important are such factors, the more opportunistic is the politicians’ behavior. Reelection concern gives therefore rise to only a partially disciplining effect on the non-congruent politician, so the society’s ability to limit the politician’s power may now dominate its ability to reelect the incumbent politician: this occurs when the higher disciplining effect of the first type of control more than offsets the selection effect of the second one.

We finally allow the society to simultaneously use both types of controls of politicians and characterize the optimal mix of them. The interplay of disciplining, selection and rent-shrinking effect gives rise to non-trivial results. In particular, we show that the introduction of a second type of control may reduce social welfare. In our model, this occurs in two cases. First, when the society’s ability to limit the politician’s power has a fully disciplining effect on the non-congruent politician, the introduction of the society’s ability not to reelect the incumbent politician reduces the effectiveness of the first type of control in disciplining politicians, and so it decreases social welfare, unless the value for the society of the selection effect of reelection more than offsets the value of the decrease in the disciplining effect. Second, the introduction of the society’s ability to limit the politician’s power may reduce the disciplining effect of reelection, and so decrease social welfare, unless the value for the society of the shrinking effect more than offsets the value of the decrease in the disciplining effect.

The role of elections in disciplining (homogenous) politicians has been well known since Barro (1973), Ferejohn (1986) and Austen-Smith and Banks (1989). Fearon (1999) and Besley (2005, 2006) inter alia argued that a second, crucial role of elections is to select politicians; this calls for considering models with heterogeneous politicians, who differ in either their motivation (Reed, 1994; Banks and Sundaram, 1998; Fearon, 1999; Besley, 2006, sec.3.3) or their competence (Berganza, 2000; Colombo, 2006). Better motivated politicians have a stronger incentive to act in the interest of the citizens, so the better the policy implemented the better the citizens’ beliefs on the politician’s motivation. The citizens will accordingly find it optimal to reelect the incumbent politician if (and only if) she implemented a sufficiently good policy. But then even badly motivated politicians may choose to implement good policies in order to be reelected. This is exactly what occurs in
our basic model, where politicians only differ in their motivation. When, however, politicians also differ in their competence, it may not be credible that the society does not reelect a high-ability politician who extracted a rent and/or reelects a low-ability politician who maximized social welfare. Reelection need not (only) depend on the policy implemented, so the disciplining effect of reelection may break down or be attenuated. This suggests that alternative types of controls of politicians could effectively replace or complement elections in achieving accountability. The present paper is a step in that direction.

The possibility that reelection concern has an undisciplining effect has recently been pointed out. Coate and Morris (1995), Maskin and Tirole (2004), Schultz (2005) and Wrasai (2005) all assume that the incumbent politician knows the true state of the world, while the citizens only know the prior probability of the different states. After observing the policy implemented, the citizens modify their beliefs on the incumbent politician’s motivation and decide whether to reelect her. It is then possible that even a well motivated politician does not implement the policy that maximizes social welfare, because this would be interpreted by the citizens as a signal that she is badly motivated, thus preventing her reelection. In this paper, there is no asymmetry of information on the social effects of the different policies, so a politician may only choose to maximize social welfare in order to be reelected. Le Borgne and Lockwood (2006) also show that elections may demotivate politicians. They consider a two-period model where a politician who is also interested in social welfare and who does not know her ability supplies in the first period a certain level of effort, which increases the probability that the only existing policy is a success, but also improves the politician’s information on her ability. This information will be valuable to the politician only in the second period; such a politician will then choose a higher level of effort when she is not exposed to the risk of not being reelected. In our model, politicians know their competence, so there is no learning effect. Finally, Colombo (2006) assumes that there exist politicians who are interested in entering history, which calls for staying in office for more than one period, never losing an electoral competition and implementing policies that will be evaluated in a positive way by the society, and shows that when politicians also differ in their competence, reelection concern may have an undisciplining effect on low-ability politicians interested in entering history. It follows that social welfare may be higher when politicians are elected for two periods than when they are elected for only one period and eventually reelected for an additional period. This cannot occur in our model, where there are not politicians interested in entering history. The introduction of the society’s ability not to reelect the incumbent politician may, however, decrease the effectiveness of other types of controls of politicians in disciplining politicians, and so reduce social welfare.

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1 Persson, Roland and Tabellini (1997) show that separation of powers may sometimes successfully perform this task.
The structure of the paper is the following. In sections 2 and 3, we compare the two types of controls of politicians when politicians, respectively, only differ in their motivation and differ in both their motivation and their competence. In section 4, we characterize the optimal mix of controls. Section 5 concludes. The proofs of all the propositions are in the Mathematical Appendix.

2 Two ways of controlling politicians

2.1 The basic principal–agent model

We consider a two–period model where, in each period, a (female) politician has to choose between policies $W$ and $R$. $W$ is the policy that maximizes social welfare, while $R$ is the policy that maximizes the politician’s rent. Without lack of generality, the utility function for the society ($S$) is such that $u_S(W) = 1$ and $u_S(R) = 0$.

There are two types of politicians. A congruent politician ($C$) is not interested in rents, but in social welfare; for convenience of exposition, we assume that she always implements policy $W$. On the other hand, a non–congruent politician ($N$) is only interested in rents; we assume that with probability $\eta$ she plays $R$, while with probability $1 - \eta$ she chooses whether to play $W$ or $R$, but actually focus on the case where $\eta$ tends to zero, i.e., the non–congruent politician almost always optimally chooses how to play. An obvious implication of the two above assumptions is that if we let $p_R$ and $p_W$ denote the probabilities the society attaches to a politician being congruent, conditional on her having played, in the first period, respectively, $R$ and $W$, then $p_W > p$, i.e., maximizing social welfare is always interpreted by the society as a (possibly very weak) signal of congrueness, and $p_R = 0$, i.e., a politician who extracts a rent reveals herself as non–congruent. Without lack of generality, the utility function for a non–congruent politician is such that $u_N(R) = 1$ and $u_N(W) = a$, with $a \in (0, 1)$; moreover, the utility from not staying in office is assumed to be equal to zero, so $a$ can be interpreted as an ego–rent and $1 - a$ as a monetary rent arising from policy $R$.

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2 We do not explicitly model an asymmetry of information between the politician and the society on the effects of the different policies on social welfare. It would however be easy to do so. For example, one can imagine that there are two equally likely states of the world, $\theta_1$ and $\theta_2$, which are only known by the politician. Let the payoffs for policy $R$ be the same as above, and assume that the politician has two additional policies available: $W_1$ and $W_2$. $W_k$ gives the same payoff as $W$ in state $\theta_k$, $k \in \{1, 2\}$, and a lower payoff (for both the society and the politician) in state $\theta_j \neq k$. This model would unfold in exactly the same way as the model in the paper. The existence of such an asymmetry of information could however explain why direct democracy might not be optimal.

3 In fact, it is easy to see that this is not really an assumption, but a property of any perfect Bayesian equilibrium.
The politician’s preferences are private information; the a priori probability that a politician is congruent is \( p \). Before being elected, a politician cannot signal her type.

Throughout all the paper, we focus on the (unique) perfect Bayesian equilibrium of the game, assuming that both the non-congruent politician and the society act in order to maximize their per-period expected utility and ruling out the zero-probability values of the parameters that give rise to multiple equilibria.

The timing of this basic model, where the society observes the policy implemented by the politician but has no available strategies, is the following.

\[
\begin{array}{c|c|c}
C & S & C \\
\hline
W & W or R & W or R \\
N \text{ plays } W or R & N \text{ plays } W or R & \text{time}
\end{array}
\]

In the second period, there is no reason why a non-congruent politician should not extract a rent. This lame-duck effect is a well known result in finitely repeated games, which will hold even when politicians also differ in their competence.

**Result 2.1** In a perfect Bayesian equilibrium, in the second period, the non-congruent politician implements (with probability one) policy \( R \), i.e., she extracts a rent.

In a two-period model where the society has no available strategies, there is no reason why a non-congruent politician should not also extract a rent in the first period. The following result, together with Result 2.1, characterizes the unique perfect Bayesian equilibrium of the game (\( n \) denotes the probability that the non-congruent politician maximizes social welfare in the first period).

**Result 2.2** In the unique perfect Bayesian equilibrium of the game, \( n = 0 \) (i.e., in the first period, the non-congruent politician extracts a rent). The per-period expected utility for the society is

\[ E(u_S) = p \quad (2.1.1) \]

In the next two subsections, we study the effects on social welfare of two ways of controlling politicians, namely the society’s ability to (partly) tie the politician’s hands and not to reelect the incumbent politician.

### 2.2 Positive or negative attitude towards a politician

The actual power of a politician depends on the society’s attitude towards her. When the society is skeptical about the will of the politician to maximize social welfare, some of its members can take a series of actions that partly tie the politician’s hands, thus making it more difficult for her both to maximize social welfare and to extract rents. Such actions are beneficial to the society whenever the politician chooses to extract a rent, whereas they are harmful when the politician acts in
order to maximize social welfare. As for the non-congruent politician, she clearly prefers having her hands free; moreover, we assume that extracting a rent continues to be her one-period dominant strategy, whatever the society’s attitude towards her.

If we let \( F \) be the society’s strategy to give the politician the freedom to implement her preferred policy and \( T \) the strategy to limit the politician’s power, to partly tie her hands, the generic strategic form of the one-shot game between a non-congruent politician and the society will be

\[
\begin{array}{c|cc}
 & F & T \\
\hline
W & a, 1 & c, e \\
R & 1, 0 & b, d \\
\end{array}
\]

where \( a, b, c, d, e \in (0, 1), e > d \) and \( b > c \).

The timing of the game is

\[
\begin{array}{ccc}
 & C \text{ plays } W & \\
N \text{ plays } W \text{ or } R & S \text{ observes } W \text{ or } R & C \text{ plays } W \\
S \text{ plays } F \text{ or } T & S \text{ plays } F \text{ or } T & \\
\end{array}
\]

For simplicity, we will henceforth assume that there is a sufficiently widespread confidence in politicians, so that in a one-shot game the society would have a positive attitude towards a politician\(^4\).

The probability that a politician plays \( W \) in the first period is at least \( p \), so at the beginning of the game the society will have a positive attitude towards the politician. As for the second period, from \( p_W > p \), the society will also show a positive attitude towards a politician who, in the first period, maximized social welfare. Finally, from \( p_R = 0 \), the society will show a negative attitude towards a politician who, in the first period, extracted a rent. These results will hold even when politician also differ in their competence.

**Result 2.3** In a perfect Bayesian equilibrium, in the first period, the society shows (with probability one) a positive attitude towards an elected politician. In the second period, the society shows (with probability one) a positive attitude towards a politician who maximized social welfare, while it shows (with probability one) a negative attitude towards a politician who extracted a rent.

\(^4\) This calls for assuming that \( p > \frac{d}{1 + e + d} \). Of course, for any \( p \), there always exist values of \( d \) and \( e \) such that the above inequality is satisfied. This assumption can therefore also be seen as a restriction on the values of the parameters \( d \) and \( e \). Finally, this assumption also implies that exactly the same results would obtain if players acted sequentially, which would seem to be the case in the institutional interpretation of this type of control (the politician implements a policy only after having known whether she is constrained to a balanced-budget).
In the second period, a non-congruent politician plays her one-period dominant strategy, i.e., she extracts a rent (Result 2.1). As for the first period, she faces a dilemma. If she extracts a rent, she will earn 1 in that period, but induce a negative attitude of the society towards her in the following period, when extracting a rent would give her only a payoff of \( b \). On the other hand, if she maximizes social welfare, she will earn only \( a \) in the first period, but guarantee herself the power to extract the full rent of 1 in the second period. The optimal behavior of the non-congruent politician depends therefore on whether \( a > b \) or \( a < b \). The following result, together with Results 2.1 and 2.3, characterizes the unique perfect Bayesian equilibrium of the game.

**Result 2.4** There exist two mutually exclusive perfect Bayesian equilibria.

(i) if \( a > b \), then \( n = 1 \) (i.e., in the first period, the non-congruent politician maximizes social welfare). The per-period expected utility for the society when it has the ability to tie (T) the politician’s hands is

\[
E(u^T_S | a>b) = p + \frac{1-p}{2}
\]  

(2.2.1)

(ii) if \( a < b \), then \( n = 0 \) (i.e., in the first period, the non-congruent politician extracts a rent). The per-period expected utility for the society is

\[
E(u^T_S | a<b) = p + \frac{(1-p)d}{2}
\]  

(2.2.2)

From eqq. (2.1.1) and (2.2.1),

\[
E(u^T_S | a>b) - E(u_S) = \frac{1-p}{2} > 0
\]

i.e., when \( a > b \), the society benefits from the first type of control of politicians. This is due to a fully disciplining effect of the society’s ability to limit the politician’s power: in the first period, a non-congruent politician chooses to maximize social welfare in order to avoid a negative attitude of the society towards her in the following period.

Moreover, from eqq. (2.1.1) and (2.2.2),

\[
E(u^T_S | a<b) - E(u_S) = \frac{(1-p)d}{2} > 0
\]

i.e., also when \( a < b \), the society benefits from the first type of control of politicians. This is due to a rent-shrinking effect: in the second period, the society shows a negative attitude towards a (non-congruent) politician who, in the first period, extracted a rent, thus limiting her ability to also extract a rent in the second period.

Finally, from eqq. (2.2.1) and (2.2.2),

\[
E(u^T_S | a>b) > E(u^T_S | a<b)
\]

i.e., from a social welfare point of view, the disciplining effect is more important than the rent-shrinking effect.
2.3 The possibility not to reelect the incumbent politician

A second way of reacting against an incumbent politician who extracted a rent is not to reelect her. In this subsection, we assume that, at the end of the first period, the society observes the policy implemented by the incumbent politician and, on the basis of this information, it chooses whether to reelect her. The society cannot commit to a reelection rule, so the timing of the game is the following.

\[
\begin{array}{cccc}
C \text{ plays } W & S \text{ observes } W \text{ or } R & S \text{ reelects the incumbent politician or elects a new one} & C \text{ plays } W \\
N \text{ plays } W \text{ or } R & & & N \text{ plays } W \text{ or } R \\
\end{array}
\]

If in the first period the incumbent politician played \( R \), from \( p_R = 0, p_R < p \), so the society will choose to never reelect a politician who extracted a rent. On the other hand, if in the first period the incumbent politician played \( W \), \( p_W > p \), so the society will always reelect a politician who maximized social welfare. A non-congruent politician anticipates that, if she plays \( W \), she will be reelected, and so earn an overall payoff of \( a + 1 \), while if she plays \( R \), she will not be reelected, and so only earn the full rent of 1. She will therefore choose to maximize social welfare.

The following result, together with result 2.1, characterizes the unique perfect Bayesian equilibrium of the game (\( s_j \) denotes the probability that the society reelects an incumbent politician who implemented policy \( j \in \{W, R\} \)).

**Result 2.5** In the unique perfect Bayesian equilibrium, \( s_W = 1 \) and \( s_R = 0 \) (i.e., the society reelects a politician only if she maximized social welfare), and \( n = 1 \) (i.e., in the first period, the non-congruent politician maximizes social welfare). The per-period expected utility for the society when it also elects (\( E \)) the politician at the beginning of the second period is

\[
E(u_S^E) = p + \frac{1-p}{2}.
\]  

(2.3.1)

From eqq. (2.1.1) and (2.3.1),

\[
E(u_S^E) - E(u_S) = \frac{1-p}{2} > 0
\]

i.e., the society also benefits from the second type of control of politicians. This is due to the (fully) disciplining effect of the society’s ability not to reelect the incumbent politician, who chooses to maximize social welfare in order to be reelected.

In equilibrium, elections give zero power, in the second period, to a politician who, in the first period, extracted a rent, so they have a stronger \(^5\) disciplining effect.

\(^5\) Throughout all the paper, we will use the terms «stronger», «dominate», «increasing» and «decreasing» as synonyms for «not-weaker», «weakly dominate», «non-decreasing» and «non-increasing». 

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than only limiting the politician’s power. From eqq. (2.2.1), (2.2.2) and (2.3.1),

\[ E\left(u^{S}_{E}\right) > E\left(u^{T}_{E} \mid a<b\right) \quad \text{and} \quad E\left(u^{S}_{E}\right) = E\left(u^{T}_{E} \mid a>b\right) \]

The disciplining effect is socially more important than the rent–shrinking effect; so, from a social welfare point of view, the society’s ability not to reelect the incumbent politician dominates its ability to limit the politician’s power. In the next section, we show that this need not however be the case when the society’s decision whether to reelect the incumbent politician depends not only on the policy she implemented in the past (as it occurred in the present section), but also on some of her intrinsic features.

3 The optimal control of politicians

3.1 The competence of a politician

We assume that politicians not only differ in their preferences (congruent vs non-congruent, C vs N), but also in their competence (high-ability vs low-ability, H vs L). There are therefore four types of politicians: CH, CL, NH and NL.

For each set of strategies, a high-ability politician adds to the society’s payoff a value \( \theta_H > 0 \), whereas a low-ability politician adds a value \( \theta_L < 0 \).

The politician’s competence is private information. The a priori probability that a politician has a high ability is \( q \), which is stochastically independent of the probability \( p \) that she is congruent.

At the end of the first period, the society observes both the policy implemented and the competence of the incumbent politician. The timing of the model without control of politicians becomes

\[ \begin{align*}
\text{CH and CL play } W & \quad \text{S observes } H \text{ or } L \\
\text{NH and NL play } W \text{ or } R & \quad \text{S observes } W \text{ or } R
\end{align*} \]

In a two-period model where the society has no available strategies, there is no reason why a non-congruent politician should not also extract a rent in the first period. The following result, together with Result 2.1, characterizes the unique perfect Bayesian equilibrium of the game (\( n_H \) and \( n_L \) denote the probabilities that, respectively, politicians NH and NL play W in the first period).

Result 3.1 In the unique perfect Bayesian equilibrium of the game, \( n_H = n_L = 0 \) (i.e., in the first period, the non-congruent politician extracts a rent, whatever her competence). If we let

\[ \hat{\theta} = q\theta_H + (1-q)\theta_L, \]

the per-period expected utility for the society is

\[ E\left(u_S\right) = \hat{\theta} + p. \quad (3.1.1) \]
In the next two subsections, we study whether and how the existence of politicians who differ in their competence affects the equilibria of the games where the society can, respectively, limit the power of the politician and choose not to reelect her.

3.2 Disciplining and rent–shrinking effects of spontaneous institutions

The timing of the game when, at the end of the first period, the society observes the competence of the politician and, also on the basis of this information, decides whether to limit her power (henceforth labelled as model $T$) is

<table>
<thead>
<tr>
<th>Time</th>
<th>$CH$ and $CL$ play $W$</th>
<th>$S$ observes $H$ or $L$</th>
<th>$NH$ and $NL$ play $W$ or $R$</th>
<th>$S$ plays $F$ or $T$</th>
</tr>
</thead>
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</tbody>
</table>

The society wishes to give the politician the freedom to act as she pleases if it thinks that she is going to maximize social welfare and to tie the hands of the politician if it thinks that she is going to extract a rent, whatever the politician’s competence. Result 2.3 continues therefore to hold, while the updated version of Result 2.4 is the following.

**Result 3.2** There exist two mutually exclusive perfect Bayesian equilibria.

(i) If $a > b$, then $n_H = n_L = 1$ (i.e., in the first period, $NH$ and $NL$ play $W$). The per–period expected utility for the society is

$$E(u_S^T | a > b) = \hat{\theta} + p + \frac{1 - p}{2} \quad (3.2.1)$$

(ii) If $a < b$, then $n_H = n_L = 0$ (i.e., in the first period, $NH$ and $NL$ play $R$). The per–period expected utility for the society is

$$E(u_S^T | a < b) = \hat{\theta} + p + \frac{(1 - p)d}{2} \quad (3.2.2)$$

Also in this case, $E(u_S^T | a > b) - E(u_S^T) = \frac{1 - p}{2} > 0$ because of a disciplining effect of the society’s ability to limit the politician’s power, and $E(u_S^T | a < b) - E(u_S^T) = \frac{(1 - p)d}{2} > 0$ because of a rent–shrinking effect.

3.3 Disciplining and selection effects of reeection

The timing of the game when the society cannot limit the power of a politician, but decides whether to reelect her (henceforth labelled as model $E$), is the following.

<table>
<thead>
<tr>
<th>Time</th>
<th>$CH$ and $CL$ play $W$</th>
<th>$S$ observes $H$ or $L$</th>
<th>$NH$ and $NL$ play $W$ or $R$</th>
<th>$S$ selects the incumbent politician or elects a new one</th>
</tr>
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The society has now two possible reasons for not reelecting the incumbent politician. As before, it may choose to replace a politician who, in the first period, played \(R\), thus revealing herself as non-congruent, with a new politician who has a probability \(p\) of being congruent. Anticipating the society’s behavior, a non-congruent politician may then choose to play \(W\) in order to be reelected. Elections may therefore still have a disciplining effect. However, the society may also choose to replace a politician who turned out to have a low level of competence with a new politician who has a probability \(q\) of being high-ability. Elections may therefore also have a selection effect.

We let \(s_{ij}\) be the probability that the society reelects a politician with ability \(i \in \{H,L\}\) who implemented policy \(j \in \{W,R\}\). The following proposition, together with result 2.1, characterizes the unique perfect Bayesian equilibrium of the game as a function of \(\theta_{H} - \theta_{L}\), i.e., the «difference in value for the society between a high-ability and a low-ability politician».

**Proposition 3.1** There exist four mutually exclusive perfect Bayesian equilibria.

(E1) If \(\theta_{H} - \theta_{L} > \max \left\{ \frac{1-p}{q}, \frac{p}{1-q} \right\} \), then \(s_{HW} = s_{HR} = 1\) (i.e., \(S\) always reelects a high-ability politician), \(s_{LW} = s_{LR} = 0\) (i.e., \(S\) never reelects a low-ability politician) and \(n_{H} = n_{L} = 0\) (i.e., in the first period, \(NH\) and \(NL\) play \(R\)). The per-period expected utility for the society is

\[
E(u_{S}^{E} | E_{1}) = \hat{\theta} + p + \frac{q(1-q)(\theta_{H} - \theta_{L})}{2} \quad (3.3.1)
\]

(E2) If \(\theta_{H} - \theta_{L} \in \left( \frac{1-p}{q}, \frac{p}{1-q} \right)\), which calls for \(p+q > 1\), then \(s_{HW} = 1\) and \(s_{HR} = 0\) (i.e., \(S\) reelects a high-ability politician only if she played \(W\)), \(s_{LW} = s_{LR} = 0\) (i.e., \(S\) never reelects a low-ability politician), \(n_{H} = 1\) and \(n_{L} = 0\) (i.e., in the first period, \(NH\) plays \(W\) and \(NL\) plays \(R\)). The per-period expected utility for the society is

\[
E(u_{S}^{E} | E_{2}) = \hat{\theta} + p + \frac{q(1-q)(\theta_{H} - \theta_{L})}{2} + \frac{(1-p)q}{2} \quad (3.3.2)
\]

(E3) If \(\theta_{H} - \theta_{L} \in \left( \frac{p}{1-q}, \frac{1-p}{1-q} \right)\), which calls for \(p+q < 1\), then \(s_{HW} = s_{HR} = 1\) (i.e., \(S\) always reelects a high-ability politician), \(s_{LW} = 1 - a\) and \(s_{LR} = 0\) (i.e., \(S\) reelects with probability \(s_{LW}\) a low-ability politician who played \(W\) and never reelects a low-ability politician who played \(R\)), \(n_{H} = 1\) and \(n_{L} = 0\) (i.e., in the first period, \(NH\) plays \(R\) and \(NL\) plays \(W\) with probability \(n_{L}\)). The per-period expected utility for the society is

\[
E(u_{S}^{E} | E_{3}) = \hat{\theta} + p + \frac{q(1-q)(\theta_{H} - \theta_{L})}{2} + \frac{p(1-q)(1-q)(\theta_{H} - \theta_{L})}{2p+q(\theta_{H} - \theta_{L})} \quad (3.3.3)
\]

(E4) If \(\theta_{H} - \theta_{L} < \min \left\{ \frac{1-p}{q}, \frac{p}{1-q} \right\}\), then \(s_{HW} = 1\) and \(s_{HR} = 0\) (i.e., \(S\) reelects a high-ability politician only if she played \(W\)), \(s_{LW} = 1 - a\) and \(s_{LR} = 0\) (i.e., \(S\) reelects with probability \(s_{LW}\) a low-ability politician who played \(W\) and never reelects a low-ability politician who played \(R\)), \(n_{H} = 1\) and \(n_{L} = 0\) (i.e., in the first period, \(NH\) plays \(R\) and \(NL\) plays \(W\) with probability \(n_{L}\)).
(i.e., in the first period, \( NH \) plays \( W \) and \( NL \) plays \( W \) with probability \( n_L \)). The per-period expected utility for the society is

\[
E(n^E_S \mid E^1) = \tilde{\theta} + p + \frac{q(1-q)(\theta_H - \theta_L)}{2} + \frac{(1-p)q}{2} + \frac{p(1-q)(1-p-q(\theta_H - \theta_L))}{p+q(\theta_H - \theta_L)} \quad (3.3.4)
\]

When \( \theta_H - \theta_L > \max \left\{ \frac{1-p}{q}, \frac{1-q}{p} \right\} \), the difference in value for the society between a high-ability and a low-ability politician is so high that the society chooses to always reelect a high-ability politician \((s_{HW} = s_{HR} = 1)\), even if she played \( R \), thus revealing herself as non-congruent, and to never reelect a low-ability politician \((s_{LW} = s_{LR} = 0)\), even if she played \( W \), thus revealing herself (in equilibrium) as congruent. A high-ability non-congruent politician anticipates that she will always be reelected, so she chooses to play \( R \) \((n_H = 0)\); a low-ability non-congruent politician anticipates that she will never be reelected, so she also plays \( R \) \((n_L = 0)\). Elections have therefore no disciplining effect. A low-ability incumbent politician is always replaced by a new politician, who has (on average) a higher competence: there is therefore the maximum selection effect.

When \( \theta_H - \theta_L < \frac{1-p}{q} \), the difference in value for the society between a high-ability and a low-ability politician is not so high to induce the society to reelect a high-ability politician who played \( R \) \((s_{HW} = 1 \text{ and } s_{HR} = 0)\), but sufficiently high to induce it to never reelect a low-ability politician \((s_{LW} = s_{LR} = 0)\), even if she played \( W \). A high-ability non-congruent politician anticipates that she will be reelected only if she plays \( W \), so she chooses to play \( W \) \((n_H = 1)\), whereas a low-ability non-congruent politician anticipates that she will never be reelected, so she plays \( R \) \((n_L = 0)\). Elections have therefore a fully disciplining effect on \( NH \) and no disciplining effect on \( NL \). Also in this case, there is the maximum selection effect.

When \( \theta_H - \theta_L < \frac{p}{1-q} \), the difference in value for the society between a high-ability and a low-ability politician is sufficiently high to induce the society to always reelect a high-ability politician \((s_{HW} = s_{HR} = 1)\), even if she played \( R \), but not so high to induce it to never reelect a low-ability politician who played \( W \) \((s_{LW} = 1 - a \text{ and } s_{LR} = 0)\) \(^6\). A high-ability non-congruent politician anticipates that she will always be reelected, so she chooses to play \( R \) \((n_H = 0)\), whereas a low-ability non-congruent politician anticipates that she will be reelected with positive probability only if she plays \( W \), so she plays \( W \) with probability \( n_L = \frac{p}{1-p} \frac{1-p-q(\theta_H - \theta_L)}{p+q(\theta_H - \theta_L)} \). Elections have therefore a partially disciplining effect on \( NL \) and no disciplining effect on \( NH \). In this case, there is less than the maximum selection effect because a low-ability politician is reelected with positive probability.

\( ^6 \) In equilibrium, the society plays a mixed strategy against a low-ability incumbent politician who maximized social welfare. Probabilities \( s_{LW} \) and \( n_L \) measure, respectively, the uncertainty of the low-ability non-congruent politician on the society’s reelection strategy and the uncertainty of the society on the strategy chosen by the low-ability non-congruent politician. See Aumann (1987) for a theoretical justification of this way of interpreting mixed strategy equilibria.
Finally, when $\theta_H - \theta_L < \min \left\{ \frac{1-p}{q}, \frac{p}{1-q} \right\}$, the difference in value for the society between a high-ability and a low-ability politician is so low to induce the society to never reelect a high-ability politician who played $R$ ($s_{HW} = 1$ and $s_{HR} = 0$) and to reelect with positive probability a low-ability politician who played $W$ ($s_{LW} = 1-a$ and $s_{LR} = 0$). A high-ability non-congruent politician anticipates that she will be reelected only if she plays $W$, so she chooses to play $W$ ($n_H = 1$), whereas a low-ability non-congruent politician anticipates that she will be reelected with positive probability only if she plays $W$, so she plays $W$ with probability $n_L = \frac{p}{1-p} \cdot \frac{1-p-q(\theta_H-\theta_L)}{1-q(\theta_H-\theta_L)}$. Elections have therefore a fully disciplining effect on $NH$ and a partially disciplining effect on $NL$. Also in this case, there is less than the maximum selection effect because a low-ability politician is reelected with positive probability.

Prop. 3.1 shows that the equilibrium that actually obtains depends on the value of $\theta_H - \theta_L$. Three properties of the different equilibria are worth mentioning. First, in all the equilibria, $s_{HW} \neq s_{LW}$ and/or $s_{HR} \neq s_{LR}$. Hence, the society’s decision whether to reelect the incumbent politician does not only depend on the policy implemented in the first period (as it occurred in the previous section, when politicians only differed in their motivation), but also on the politician’s competence, i.e., a factor that is not under the control of the politician herself. Second, $n_H < 1$ (in fact, $n_H = 0$) for all $\theta_H - \theta_L > \frac{1}{1-q}$ and $n_L < 1$ for all $\theta_H - \theta_L$. Hence, the existence of factors that are not under the control of the politicians but that affect their probability of being reelected reduces the politicians’ incentive to maximize social welfare. Third, both $n_H$ and $n_L$ are decreasing in $\theta_H - \theta_L$. Hence, the more important are the aforementioned factors, the more opportunistic is the politicians’ behavior.

3.4 The best way of controlling politicians

In the model we analyzed in section 2, the society’s ability to limit the politician’s power gave rise to either a disciplining or a rent-shrinking effect, whereas its ability not to reelect the incumbent politician always gave rise to a disciplining effect. The disciplining effect was socially more important than the rent-shrinking effect; so, from a social welfare point of view, the society’s ability not to reelect the incumbent politician dominated its ability to limit the politician’s power.

When politicians also differ in their competence, a second positive effect of re-election arises: the selection effect. One could thus conjecture that the society’s ability not to reelect the incumbent politician should a fortiori dominate its ability to limit the politician’s power. In fact, this need not be the case because the introduction of a motive for re-election that is not under the control of the politician may reduce the politician’s incentive to maximize social welfare in order to be re-elected.

In this subsection, we compare the effectiveness of the two ways of controlling politicians in maximizing social welfare. According to Result 3.2 and Prop. 3.1, the effects of the society’s ability to limit the politician’s power (model $T$) depend
in a crucial way on whether \( a > b \) or \( a < b \), while the effects of the society’s ability not to reelect the incumbent politician (model \( E \)) depend on the value of \( \theta_H - \theta_L \).

We will now consider separately the cases where \( a > b \) and \( a < b \) and, for each of them, find the values of \( \theta_H - \theta_L \) such that, respectively, \( E \succ T \) and \( T \succ E \).

Let firstly \( a > b \). The society’s ability to limit the politician’s power has a fully disciplining effect on the non-congruent politician, while it gives rise to no rent-shrinking effect. On the other hand, the society’s ability not to reelect the incumbent politician has (at most) a partially disciplining effect, but it gives rise to a positive selection effect. Hence, \( E \succ T \) if (and only if) the equilibrium value of the selection effect under \( E \) exceeds the value of the higher disciplining effect under \( T \).

A rise in \( \theta_H - \theta_L \) increases the equilibrium value of the selection effect under \( E \) but, by decreasing the disciplining effect, it also increases the higher disciplining effect under \( T \). Hence, the society’s preference between \( E \) and \( T \) need not be monotonic in \( \theta_H - \theta_L \).

**Proposition 3.2** If \( a > b \), then \( E \succ T \) if (and only if) either \( \theta_H - \theta_L \in (\frac{1-p}{q-p}, \frac{p}{1-p}) \) or \( \theta_H - \theta_L > \max\{\frac{1-p}{q(1-q)}, \frac{p}{1-p}\} \). \(^7\)

Let now \( a < b \). The society’s ability to limit the politician’s power gives rise to a positive rent-shrinking effect, while it has no disciplining effect. Hence, \( E \succ T \) if (and only if) the sum of the equilibrium values of the selection and (at most partial) disciplining effect under \( E \) exceeds the value of the rent-shrinking effect under \( T \). In our model, the value of the rent-shrinking effect is lower than the value of the (full) disciplining effect, so, with respect to the case where \( a > b \), it must now be relatively more likely that the society prefers \( E \) to \( T \). In fact, this is always the case.

**Proposition 3.3** If \( a < b \), then \( E \succ T \).

In the next section, we study how the two ways of controlling politicians complement each other. In particular, we compare the model where the society can both limit the politician’s power and choose not to reelect her with the ones where it can use only one of the two types of controls.

\(^7\) There is only a zero-probability set of values of the parameters such that \( E \sim T \), so in all the other cases \( T \succ E \) almost always.
4 The optimal mix of controls of politicians

4.1 The simultaneous use of the two ways of controlling politicians

In the model with no control of politicians we analyzed in section 3.1, a non-congruent politician always extracts a rent; furthermore, a low-ability politician always stays in office for two periods. There are therefore two problems for the society: opportunistic behavior and bad politicians.

The society’s ability to limit the politician’s power weakens the first type of problem. In particular, if $a > b$, a non-congruent politicians maximizes social welfare in the first period, while if $a < b$, the society is able to reduce the loss in the second period coming from a non-congruent politician who extracts a rent. On the other hand, the ability to choose whether to reelect the incumbent politician weakens the second type of problem, since it gives the society the opportunity to replace a politician who turned out to have a low level of competence with a new politician, who has on average a higher competence. It may now be useful to study whether the society can benefit from the combined effect of the two types of controls of politicians.

The timing of the more complete game (henceforth labelled as model $C$), where the society can both limit the power of the politician and choose not to reelect her, is the following.

The following two propositions, together with results 2.1 and 2.3, characterize the unique perfect Bayesian equilibrium of the game when, respectively, $a > b$ and $a < b$.

**Proposition 4.1** If $a > b$, there exist four mutually exclusive perfect Bayesian equilibria.

(C1) If $\theta_H - \theta_L > \max \left\{ \frac{1-p}{q}, \frac{p-d}{1-q} \right\}$, then $s_{HW} = s_{HR} = 1$ (i.e., $S$ always reelects a high-ability politician), $s_{LW} = s_{LR} = 0$ (i.e., $S$ never reelects a low-ability politician), $n_H = 1$ and $n_L = 0$ (i.e., in the first period, $NH$ plays $W$ and $NL$ plays $R$).

The per-period expected utility for the society is

$$
E(u_C^S | a>b, C_1=C_2) = \hat{\theta} + p + \frac{q(1-q)(\theta_H - \theta_L)}{2} + \frac{(1-p)q}{2} \tag{4.1.1}
$$

$^8$ In fact, if the difference in value for the society between a high-ability and a low-ability politician does not exceed a threshold, reelection concern has a (partially) disciplining effect on the non-congruent politician, so also the first type of problem is weakened.
(C2) If $\theta_H - \theta_L \in \left(\frac{1-p}{q}, \frac{p-d}{q}\right)$, which calls for $p + q(1-d) > 1$, then $s_{HW} = 1$ and $s_{HR} = 0$ (i.e., $S$ reelects a high-ability politician only if she played $W$), $s_{LW} = s_{LR} = 0$ (i.e., $S$ never reelects a low-ability politician), $n_H = 1$ and $n_L = 0$ (i.e., in the first period, NH plays $W$ and NL plays $R$). The per-period expected utility for the society is $E(u^C_S |_{a>b,C1=C2})$:

(C3) If $\theta_H - \theta_L \in \left(\frac{1-p}{q}, \frac{p-d}{q}\right)$, which calls for $p+q(1-d) < 1$, then $s_{HW} = s_{HR} = 1$ (i.e., $S$ always reelects a high-ability politician), $s_{LW} = 1 - a$ and $s_{LR} = 0$ (i.e., $S$ reelects with probability $s_{LW}$ a low-ability politician who played $W$ and never reelects a low-ability politician who played $R$), $n_H = 1$ and $n_L = \frac{p}{1-p} - \frac{p-q(\theta_H - \theta_L)}{q(\theta_H - \theta_L)}$ (i.e., in the first period, NH plays W and NL plays W with probability $n_L$). The per-period expected utility for the society is

$$E(u^C_S |_{a>b,C3=C4}) = \hat{\theta} + p + \frac{q(1-q)(\theta_H - \theta_L)}{2} + \frac{(1-p)q}{2} + \frac{p(1-q)(1-p-q(\theta_H - \theta_L))}{2}$$

(C4) If $\theta_H - \theta_L < \min\left\{\frac{1-p}{q}, \frac{p-d}{q}\right\}$, then $s_{HW} = 1$ and $s_{HR} = 0$ (i.e., $S$ reelects a high-ability politician only if she played $W$), $s_{LW} = 1 - a$ and $s_{LR} = 0$ (i.e., $S$ reelects with probability $s_{LW}$ a low-ability politician who played W and never reelects a low-ability politician who played $R$), $n_H = 1$ and $n_L = \frac{p}{1-p} - \frac{1-p-q(\theta_H - \theta_L)}{q(\theta_H - \theta_L)}$ (i.e., in the first period, NH plays $W$ and NL plays $W$ with probability $n_L$). The per-period expected utility for the society is $E(u^C_S |_{a>b,C3=C4})$.

When $a > b$, a non-congruent politician who knew that she would be reelected would play $W$ in the first period in order to avoid a negative attitude of the society towards her in the following period. A high-ability politician who played $W$ in the first period would always be reelected ($s_{HW} = 1$), so it is actually optimal for a high-ability non-congruent politician to play $W$ ($n_H = 1$). On the other hand, a low-ability politician who played W in the first period might not be reelected ($s_{LW} < 1$), so in equilibrium a low-ability non-congruent politician plays $R$ with positive probability ($n_L < 1$). The combined use of the two types of controls has therefore a fully disciplining effect on the high-ability non-congruent politician, but (at most) a partially disciplining effect on the low-ability non-congruent politician.

**Proposition 4.2** If $a < b$, there exist four mutually exclusive perfect Bayesian equilibria.

(C1) If $\theta_H - \theta_L > \max\left\{\frac{1-p}{q}, \frac{p-d}{q}\right\}$, then $s_{HW} = s_{HR} = 1$ (i.e., $S$ always reelects a high-ability politician), $s_{LW} = s_{LR} = 0$ (i.e., $S$ never reelects a low-ability politician) and $n_H = n_L = 0$ (i.e., in the first period, NH and NL play $R$). The per-period expected utility for the society is

$$E(u^C_S |_{a<b,C1}) = \hat{\theta} + p + \frac{q(1-q)(\theta_H - \theta_L)}{2} + \frac{(1-p)q}{2}$$

(C2) If $\theta_H - \theta_L \in \left(\frac{1-p}{q}, \frac{p-d}{q}\right)$, which calls for $p + q(1-d) > 1$, then $s_{HW} = 1$ and $s_{HR} = 0$ (i.e., $S$ reelects a high-ability politician only if she played $W$), $s_{LW} =$
low—ability non—congruent politician and either a fully or no disciplining effect on the high—ability non—congruent politician. The per—period expected utility for the society is

$$E(u^C_{S|a<b,C2}) = \hat{\theta} + p + \frac{q(1-q)(\theta_H - \theta_L)}{2} + \frac{(1-p)q}{2}$$  \hspace{1em} (4.1.4)

(C3) If $\theta_H - \theta_L \in \left(\frac{p-d}{q}, \frac{1-p}{q}\right)$, which calls for $p+q(1-d) < 1$, then $s_{HW} = s_{HR} = 1$ (i.e., $S$ always reelects a high—ability politician), $s_{LW} = 1-a$ and $s_{LR} = 0$ (i.e., $S$ reelects with probability $s_{LW}$ a low—ability politician who played $W$ and never reelects a low—ability politician who played $R$), $n_H = 0$ and $n_L = \frac{p}{1-p} \frac{1-p-q(\theta_H - \theta_L)}{p+q(\theta_H - \theta_L)}$ (i.e., in the first period, $NH$ plays $R$ and $NL$ plays $W$ with probability $n_L$). The per—period expected utility for the society is

$$E(u^C_{S|a<b,C3}) = \hat{\theta} + p + \frac{q(1-q)(\theta_H - \theta_L)}{2} + \frac{(1-p)q}{2} + \frac{p(1-q)(\theta_H - \theta_L)}{p+q(\theta_H - \theta_L)}$$  \hspace{1em} (4.1.5)

(C4) If $\theta_H - \theta_L < \min\left\{\frac{1-p}{q}, \frac{p-d}{1-q}\right\}$, then $s_{HW} = 1$ and $s_{HR} = 0$ (i.e., $S$ reelects a high—ability politician only if she played $W$), $s_{LW} = 1-a$ and $s_{LR} = 0$ (i.e., $S$ reelects with probability $s_{LW}$ a low—ability politician who played $W$ and never reelects a low—ability politician who played $R$), $n_H = 1$ and $n_L = \frac{p}{1-p} \frac{1-p-q(\theta_H - \theta_L)}{p+q(\theta_H - \theta_L)}$ (i.e., in the first period, $NH$ plays $W$ and $NL$ plays $W$ with probability $n_L$). The per—period expected utility for the society is

$$E(u^C_{S|a<b,C4}) = \hat{\theta} + p + \frac{q(1-q)(\theta_H - \theta_L)}{2} + \frac{(1-p)q}{2} + \frac{p(1-q)(\theta_H - \theta_L)}{p+q(\theta_H - \theta_L)}$$  \hspace{1em} (4.1.6)

When $a < b$, a non—congruent politician who knew that she would be reelected would play $R$ in the first period. When $\theta_H - \theta_L > \frac{p-d}{q}$, a high—ability non—congruent politician anticipates that she will always be reelected ($s_{HW} = s_{HR} = 1$), so she actually chooses to play $R$ ($n_H = 0$), while when $\theta_H - \theta_L < \frac{p-d}{q}$, she anticipates that she will be reelected only if she plays $W$ ($s_{HW} = 1$ and $s_{HR} = 0$), so she chooses to play $W$ ($n_H = 1$). A low—ability politician who played $W$ in the first period might not be reelected ($s_{LW} < 1$), so in equilibrium a low—ability non—congruent politician plays $R$ with positive probability ($n_L < 1$). The combined use of the two types of controls has therefore (at most) a partially disciplining effect on the low—ability non—congruent politician and either a fully or no disciplining effect on the high—ability non—congruent politician.

### 4.2 Two controls are not always better than one

In this subsection, we compare social welfare in the perfect Bayesian equilibria of the games where the society controls politicians in either one or two ways. We will consider separately the cases where $a > b$ and $a < b$ and, for each of them, find the values of $\theta_H - \theta_L$ such that the society benefits from the combined use of the two types of controls.

Let us firstly compare social welfare under $T$ and $C$. 

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If \( a > b \), according to Prop. 3.2, the society’s ability to limit the politician’s power has a fully disciplining effect on the non–congruent politician, who chooses to maximize social welfare in order to avoid a negative attitude of the society towards her in the following period. When the society also has the ability not to reelect the incumbent politician, a low–ability non–congruent politician anticipates that she might not be reelected even if she maximizes social welfare, so she is less concerned with the (eventual) future negative attitude of the society towards her, and she might thus choose to extract a rent. On the other hand, in equilibrium the society replaces with positive probability a low–ability politician with a new politician, who has on average a higher competence, so there is a positive selection effect. Hence, \( \mathcal{C} \succ \mathcal{T} \) if (and only if) the equilibrium value for the society of the selection effect under \( \mathcal{C} \) exceeds the value of the higher disciplining effect under \( \mathcal{T} \). The following proposition shows that this occurs when the difference in value for the society between a high–ability and a low–ability politician exceeds a threshold.

**Proposition 4.3** If \( a > b \), then \( \mathcal{C} \succ \mathcal{T} \) if (and only if) \( \theta_H - \theta_L > \frac{1-p}{q} \).

If \( a < b \), according to Prop. 3.2, the society’s ability to limit the politician’s power has no disciplining effect, but a rent–shrinking effect. When the society also has the ability not to reelect the incumbent politician, a non–congruent politician might choose to maximize social welfare in order to be reelected. Moreover, in equilibrium the society replaces with positive probability a low–ability politician with a new politician, who has on average a higher competence, so there is a positive selection effect. Hence, \( \mathcal{C} \succ \mathcal{T} \) if (and only if) the sum of the equilibrium values for the society of the selection and disciplining (if any) effect under \( \mathcal{C} \) exceeds the value of the rent–shrinking effect under \( \mathcal{T} \). In fact, this is always the case.

**Proposition 4.4** If \( a < b \), then \( \mathcal{C} \succ \mathcal{T} \).

Let us compare now social welfare under \( \mathcal{E} \) and \( \mathcal{C} \).

If \( a > b \), according to Prop. 4.1, in equilibrium the society never reelects a politician who extracted a rent, so the value of the rent–shrinking effect under \( \mathcal{C} \) is equal to zero. The preference between \( \mathcal{E} \) and \( \mathcal{C} \) must therefore depend on the equilibrium values of the disciplining (if any) and selection effect. Comparing Prop. 3.1 and 4.1, one can easily see that the equilibrium behavior of the different types of politicians is the same under both \( \mathcal{E} \) and \( \mathcal{C} \) except when the difference in value for the society between a high–ability and a low–ability politician exceeds a threshold. In such a case, under both \( \mathcal{E} \) and \( \mathcal{C} \) a high–ability politician is always reelected, whatever the policy implemented, but while under \( \mathcal{E} \) a high–ability non–congruent politician extracts a rent, under \( \mathcal{C} \) she maximizes social welfare (in order to avoid a negative attitude of the society towards her in the following period), so

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9. \( \mathcal{C} \sim \mathcal{T} \) only when \( \theta_H - \theta_L = \frac{1-p}{q} \), so in all the other cases \( \mathcal{T} \succ \mathcal{C} \).
the society actually benefits from the ability to also limit the power of the incumbent politician. This result is formalized in the following proposition.

**Proposition 4.5** If \(a > b\), then \(C \succeq E\), and \(C \succ E\) if (and only if) \(\theta_H - \theta_L > \frac{p}{1-q}\).

If \(a < b\), the result is less straightforward because the society’s ability to also limit the politician’s power may give rise to either a direct (positive) effect or an indirect (negative) effect. The direct effect obtains when under both \(E\) and \(C\) the society reelects a high-ability politician who extracted a rent; in such a case, under \(C\) the society benefits from a rent-shrinking effect. On the other hand, the indirect (or strategic) effect obtains when under \(E\) the society chooses not to reelect a high-ability politician who extracted a rent, while under \(C\) it chooses to reelect such a politician, because it is now able to limit the rent she is going to extract. In such a case, under \(E\) a high-ability non-congruent politician will choose to maximize social welfare (in order to be reelected), while under \(C\) she will extract a rent, so under \(E\) the society would benefit from a higher disciplining effect. The direct effect obtains when \(\theta_H - \theta_L > \frac{p}{1-q}\) (and, hence, \(C \succeq E\)), while the indirect effect obtains when \(\theta_H - \theta_L \in \left(\frac{p-d}{1-q}, \frac{p}{1-q}\right)\) (and, hence, \(E \succ C\)). Finally, when \(\theta_H - \theta_L < \frac{p-d}{1-q}\), under both \(E\) and \(C\) the high-ability non-congruent politician maximizes social welfare (in order to be reelected): \(E\) and \(C\) have therefore the same disciplining effect; moreover, under \(C\) there is no rent-shrinking effect (and, hence, \(E \sim C\)).

**Proposition 4.6** If \(a < b\), then \(C \succ E\) if (and only if) \(\theta_H - \theta_L > \frac{p}{1-q}\), \(E \succ C\) if (and only if) \(\theta_H - \theta_L \in \left(\frac{p-d}{1-q}, \frac{p}{1-q}\right)\), and \(C \sim E\) if (and only if) \(\theta_H - \theta_L > \max\{\frac{1-p}{q}, \frac{p}{1-q}\}\).

### 4.3 The optimal mix of controls

**Prop. 3.2, 4.3 and 4.5** provided a pairwise social welfare comparison between models \(T\), \(E\) and \(C\) when \(a > b\). As an immediate corollary, we obtain the set \(\Omega^*(a > b) \subseteq \{T, E, C\}\) of controls of politicians that maximize social welfare.

**Corollary 4.1**

\[
\Omega^*(a > b) = \begin{cases} 
T & \text{if } \theta_H - \theta_L < \frac{1-p}{q} \\
\{E, C\} & \text{if } \theta_H - \theta_L \in \left(\frac{1-p}{q}, \frac{p}{1-q}\right) \\
C & \text{if } \theta_H - \theta_L > \max\{\frac{1-p}{q}, \frac{p}{1-q}\}
\end{cases}
\]

According to this corollary, when spontaneous institutions have a good disciplining effect on the incumbent politician \((a > b)\), it is optimal for the society not to have frequent elections, unless the competence of politicians is very important \((\theta_H - \theta_L > \frac{1-p}{q})\).

Finally, as a corollary of **Prop. 3.3, 4.4 and 4.6**, we obtain the set \(\Omega^*(a < b) \subseteq \{T, E, C\}\) of controls of politicians that maximize social welfare when \(a < b\).
Corollary 4.2

\[
\Omega^*(a < b) = \begin{cases} 
\{E, C\} & \text{if } \theta_H - \theta_L < \frac{p-d}{1-q} \\
E & \text{if } \theta_H - \theta_L \in \left(\frac{p-d}{1-q}, \frac{p}{1-q}\right) \\
C & \text{if } \theta_H - \theta_L > \frac{p}{1-q}
\end{cases}
\]

According to this corollary, the existence of spontaneous institutions that only have a rent–shrinking effect \((a < b)\) either reduce or do not affect social welfare, unless the competence of politicians is very important \((\theta_H - \theta_L > \frac{p}{1-q})\).

5 Conclusion

Modern democracies are characterized by spontaneous reactions by the citizens to the incumbent politician’s behavior. Consider for example anti–government demonstrations. They aim at (somehow) affecting the politician’s future behavior: a politician who is continuously and massively criticized by the public opinion will indeed find it difficult to implement her preferred policies. But then a politician may deem it optimal to act more in line with the general interest (in order to avoid anti–government demonstrations) than she would do in a less democratic society.

Modern democracies are also characterized by frequent elections. A politician who pursues her own interests may find it difficult to be reelected. But then a politician may deem it optimal to act more in line with the general interest (in order to be reelected) than she would do in a less democratic society.

The two types of controls of politicians analyzed in this paper would seem therefore to complement each other in disciplining politicians. In fact, this need not be the case, because they differ in an important respect: while the first type of control only tends to “punish” the incumbent politician’s bad behavior in the past in order to improve her behavior in the future, the second type of control tends to both “punish” the incumbent politician’s bad behavior in the past (which is a signal of bad motivation, also for the future periods) and “reward” the incumbent politician’s good competence (good competence in the past also implies good competence in the future). Reelection may therefore crucially depend on some intrinsic and non–controllable features of a politician (a sort of trademark of many modern democracies). But then a politician who anticipates that it is sufficiently likely that she will not be reelected even if she implements the policies that maximize social welfare will choose to pursue her own interests. It follows that in democracies characterized by strong spontaneous institutions and where the personal characteristics of a politician are considered as sufficiently important, too frequent elections reduce the incentive of politicians to pursue the general interest, and may even reduce social welfare.
References


Mathematical Appendix

Proof of Proposition 3.1

The first step of the proof calls for obtaining the society’s optimal reelection strategy \( s_{ij}, i \in \{H, L\}, j \in \{W, R\} \), given the strategies of the different types of politicians.

Let \( Eu_S(2, ij) \) and \( Eu_S(2, new) \) be the society’s expected utility in the second period when it, respectively, reelects a politician \( i \in \{H, L\} \) who implemented policy \( j \in \{W, R\} \) and elects a new politician.

From

\[
Eu_S(2, HW) - Eu_S(2, new) = \theta_H + \frac{p}{p + (1 - p)n_H} - \hat{\theta} - p > 0
\]

we have that

\[
s_{HW} = 1.
\] (A.1)

From

\[
Eu_S(2, HR) - Eu_S(2, new) = \theta_H - \hat{\theta} - p
\]

we have that \(^{10}\)

\[
s_{HR} = \begin{cases} 
1 & \text{if } \theta_H - \theta_L > \frac{p}{1-q} \\
0 & \text{if } \theta_H - \theta_L < \frac{p}{1-q}
\end{cases}
\] (A.2)

From

\[
Eu_S(2, LW) - Eu_S(2, new) = \theta_L + \frac{p}{p + (1 - p)n_L} - \hat{\theta} - p
\]

we have that

\[
s_{LW} = \begin{cases} 
1 & \text{if } \theta_H - \theta_L < \frac{1}{q} \left(\frac{p}{p + (1 - p)n_L} - p\right) \\
0 & \text{if } \theta_H - \theta_L > \frac{1}{q} \left(\frac{p}{p + (1 - p)n_L} - p\right) \\
[0, 1] & \text{if } \theta_H - \theta_L = \frac{1}{q} \left(\frac{p}{p + (1 - p)n_L} - p\right)
\end{cases}
\] (A.3)

Finally, from

\[
Eu_S(2, LR) - Eu_S(2, new) = \theta_L - \hat{\theta} - p < 0
\]

we have that

\[
s_{LR} = 0.
\] (A.4)

In a perfect Bayesian equilibrium, both types of non–congruent politicians play a best response to the society’s reelection strategy \( s_{ij}, i \in \{H, L\}, j \in \{W, R\} \). The second step of the proof calls therefore for obtaining the best response correspondences for the two types of non–congruent politicians.

\(^{10}\) Notice that when \( \theta_H - \theta_L = \frac{p}{1-q} \), all \( s_{HR} \in [0, 1] \) are optimal. We will henceforth always neglect this case, as well as all other zero–probability sets of parameters that either give rise to multiple equilibria or simply complicate the notation.
Let \( E_{u_{Ni}}(j, s_{ij}) \) be the per-period expected utility of an incumbent politician \( Ni, i \in \{H, L\} \) when she implements policy \( j \in \{W, R\} \), given the society’s reelection strategy \( s_{ij}, i \in \{H, L\}, j \in \{W, R\} \).

From
\[
E_{u_{NH}}(W, s_{HW}) - E_{u_{NH}}(R, s_{HR}) = \frac{a + s_{HW}}{2} - \frac{1 + s_{HR}}{2},
\]
taking into account that \( s_{HW} = 1 \) (eq. A.1), we have that
\[
n_H = \begin{cases} 
1 & \text{if } s_{HR} < a \\
0 & \text{if } s_{HR} > a \\
[0, 1] & \text{if } s_{HR} = a
\end{cases}
\]  
(A.5)

From
\[
E_{u_{NL}}(W, s_{LW}) - E_{u_{NL}}(R, s_{LR}) = \frac{a + s_{LW}}{2} - \frac{1 + s_{LR}}{2},
\]
taking into account that \( s_{LR} = 0 \) (eq. A.4), we have that
\[
n_L = \begin{cases} 
1 & \text{if } s_{LW} > 1 - a \\
0 & \text{if } s_{LW} < 1 - a \\
[0, 1] & \text{if } s_{LW} = 1 - a
\end{cases}
\]  
(A.6)

The final step of the proof works as follows. For each reelection strategy \( s_{ij} \), \( i \in \{H, L\}, j \in \{W, R\} \), using eqq. A.5–A.6 we compute the optimal policy to be implemented by the two types of non-congruent politicians; then we use eqq. A.2–A.3 to check whether the assumed reelection strategy is actually optimal, i.e., it is sequentially rational. With this procedure, we compute all the perfect Bayesian equilibria of the game.

Let firstly \( s_{HR} = 1 \) and \( s_{LW} = 0 \). From eqq. A.5–A.6, \( n_H = n_L = 0 \). Hence, from eq. A.2, we actually have that \( s_{HR} = 1 \) if
\[
\theta_H - \theta_L > \frac{p}{1 - q}
\]  
(A.7)

and, from eq. A.3, \( s_{LW} = 0 \) if
\[
\theta_H - \theta_L > \frac{1 - p}{q}.
\]  
(A.8)

If eqq. A.7 and A.8 are satisfied, there exists therefore a perfect Bayesian equilibrium with \( s_{HW} = 1, s_{HR} = 1, s_{LW} = 0, s_{LR} = 0 \) and \( n_H = n_L = 0 \) (equilibrium \( E1 \) in prop. 3.1). The per-period expected utility for the society is
\[
E(u_S^{E1}) = \frac{\hat{\theta} + p + q(\theta_H + p) + (1 - q)(\hat{\theta} + p)}{2}
\]
which can be rewritten as eq. 3.3.1.

Let now \( s_{HR} = 0 \) and \( s_{LW} = 1 \). From eqq. A.5–A.6, \( n_H = n_L = 1 \). Hence, from eq. A.2, we actually have that \( s_{HR} = 0 \) if
\[
\theta_H - \theta_L < \frac{p}{1 - q}
\]  
(A.9)
and, from eq. A.3, \( s_{LW} = 1 \) if

\[ \theta_H - \theta_L \leq 0. \quad (A.10) \]

Eq. A.10 is never satisfied, so there cannot exist a perfect Bayesian equilibrium with \( s_{HW} = 1, s_{HR} = 0, s_{LW} = 1, s_{LR} = 0 \) and \( n_H = n_L = 1 \). The intuition is straightforward. Suppose that the society reelects all politicians and only politicians who maximized social welfare. A non-congruent politician would accordingly always choose to maximize social welfare in order to be reelected, so the probability that a politician is congruent conditional on her having maximized social welfare would tend to \( p \). Maximizing social welfare would therefore be a very weak signal of congruence, so it would not be optimal for the society to actually reelect a low-incumbent politician who maximized social welfare. This suggests that, in equilibrium, \( s_{LW} > 0 \) only when the society plays a mixed strategy.

Let therefore \( s_{HR} = 0 \) and \( s_{LW} \in (0,1) \). From eq. A.5, \( n_H = 1 \). As for \( n_L \), we have to consider three cases:

(i) if \( s_{LW} > 1 - a \), from eq. A.6, \( n_L = 1 \). Hence, from eq. A.3, \( s_{LW} \in (0,1) \) calls for

\[ \theta_H - \theta_L = 0 \]

which never occurs.

(ii) if \( s_{LW} = 1 - a \), from eq. A.6, \( n_L \in [0,1] \). Hence, from eq. A.3, \( s_{LW} \in (0,1) \) calls for

\[ n_L = \frac{p}{1-p} \frac{1 - p - q (\theta_H - \theta_L)}{p + q (\theta_H - \theta_L)} \]

The constraint \( n_L \geq 0 \) calls for

\[ \theta_H - \theta_L \leq \frac{1 - p}{q} \quad (A.11) \]

whereas the constraint \( n_L \leq 1 \) calls for

\[ \theta_H - \theta_L \geq 0 \]

which is always satisfied. Finally, from eq. A.2, \( s_{HR} = 0 \) is actually optimal if

\[ \theta_H - \theta_L < \frac{p}{1-q}. \quad (A.12) \]

If eqq. A.11–A.12 are satisfied, there exists therefore a perfect Bayesian equilibrium with \( s_{HW} = 1, s_{HR} = 0, s_{LW} = 1 - a, s_{LR} = 0, n_H = 1 \) and \( n_L = \frac{p}{1-p} \frac{1 - p - q (\theta_H - \theta_L)}{p + q (\theta_H - \theta_L)} \) (equilibrium \( E3 \) in Prop. 3.1). The per-period expected utility for the society is \(^{11}\)

\[ E \left( u^E_S | E3 \right) = \frac{\hat{\theta} + p + (1-p)(1-q) n_L + q (\theta_H + p) + (1-q)(\hat{\theta} + p)}{2} \]

\(^{11}\) In the mixed strategies equilibrium under consideration, the society is indifferent between reelecting and not reelecting a low-ability incumbent politician who maximized social welfare. We can therefore compute its expected utility by assuming that it does not reelect such a politician with probability one.
which, after straightforward algebraic manipulations, can be rewritten as EQ. 3.3.3. (iii) if \( s_{LW} < 1 - a \), from EQ. A.6, \( n_L = 0 \). Hence, from EQ. A.3, \( s_{LW} \in (0,1) \) calls for

\[ \theta_H - \theta_L = \frac{1-p}{q} \]

which occurs with probability zero. We can thus rule out this case.

Using exactly the same procedure for all the other reelection strategies \( s_{ij} \in [0,1] \), \( i \in \{H, L\}, j \in \{W, R\} \), with \( s_{HW} = 1 \) and \( s_{LR} = 0 \), one can easily find all the other equilibria. Equilibria \( E1 - E4 \) represent therefore all the perfect Bayesian equilibria of the game. The sets of parameters that give rise to the different equilibria are disjoint, so for a given set of parameters the game has a unique perfect Bayesian equilibrium. ■

**Proof of proposition 3.2**

For a given value of \( \theta_H - \theta_L \), equilibrium \( E_l, l \in \{1, \ldots, 4\} \) obtains, and \( \mathcal{E} \succ \mathcal{T} \) if (and only if) \( E(u_S^E | E_l) > E(u_S^T | E_l) \).

Let firstly \( \theta_H - \theta_L < \min \left\{ \frac{1-p}{q}, \frac{p}{1-q} \right\} \). From

\[ E(u_S^E | E_l) - E(u_S^T | E_l) = \frac{q(1-q)(\theta_H - \theta_L)}{2} + \frac{p(1-q)}{2} \frac{1-p-q(\theta_H - \theta_L)}{p+q(\theta_H - \theta_L)} - \frac{(1-p)(1-q)}{2} \]

after straightforward algebraic manipulations, \( E(u_S^E | E_4) - E(u_S^T | E_4) > 0 \) if (and only if) \( \theta_H - \theta_L > \frac{1-p}{q} \). Hence, \( \theta_H - \theta_L < \min \left\{ \frac{1-p}{q}, \frac{p}{1-q} \right\} \Rightarrow \mathcal{T} \succ \mathcal{E} \).

Let now \( \theta_H - \theta_L \in \left( \frac{p}{1-q}, \frac{1-p}{q} \right) \). We have seen above that, if \( \theta_H - \theta_L < \frac{1-p}{q} \), \( E(u_S^E | E_4) - E(u_S^T | E_4) < 0 \). Taking into account that, for all \( \theta_H - \theta_L \), \( E(u_S^E | E_4) > E(u_S^T | E_3) \), we have that \( E(u_S^E | E_3) - E(u_S^T | E_3) < 0 \). Hence, \( \theta_H - \theta_L \in \left( \frac{p}{1-q}, \frac{1-p}{q} \right) \Rightarrow \mathcal{T} \succ \mathcal{E} \).

Let now \( \theta_H - \theta_L \in \left( \frac{p}{1-q}, \frac{1-p}{q} \right) \). From

\[ E(u_S^E | E_2) - E(u_S^T | E_2) = \frac{q(1-q)(\theta_H - \theta_L)}{2} + \frac{p(1-q)}{2} \frac{1-p-q(\theta_H - \theta_L)}{p+q(\theta_H - \theta_L)} - \frac{(1-p)(1-q)}{2} \]

\( \mathcal{E} \succ \mathcal{T} \) if (and only if) \( \theta_H - \theta_L > \frac{1-p}{q} \). Hence, \( \theta_H - \theta_L \in \left( \frac{p}{1-q}, \frac{1-p}{q} \right) \Rightarrow \mathcal{E} \succ \mathcal{T} \).

Finally, let \( \theta_H - \theta_L > \max \left\{ \frac{p}{1-q}, \frac{1-p}{q} \right\} \). From

\[ E(u_S^E | E_1) - E(u_S^T | E_1) = \frac{q(1-q)(\theta_H - \theta_L)}{2} + \frac{p(1-q)}{2} \frac{1-p-q(\theta_H - \theta_L)}{p+q(\theta_H - \theta_L)} - \frac{(1-p)(1-q)}{2} \]

\( \mathcal{E} \succ \mathcal{T} \) if (and only if) \( \theta_H - \theta_L > \frac{1-p}{q(1-q)} \). Taking into account that \( \frac{1-p}{q(1-q)} > \frac{1-p}{q} \), we have that \( \theta_H - \theta_L > \max \left\{ \frac{p}{1-q}, \frac{1-p}{q(1-q)} \right\} \Rightarrow \mathcal{E} \succ \mathcal{T} \). ■
Proof of Proposition 3.3

For a given value of $\theta_H - \theta_L$, equilibrium $E_l$, $l \in \{1, \ldots, 4\}$ obtains, and $E \succ T$ if (and only if) $E(u^E_{SL}|E_l) > E(u^T_{SL}|a<b)$.

Let firstly $\theta_H - \theta_L < \min \left\{ \frac{p-1}{q}, \frac{p}{1-q} \right\}$. In equilibrium, the society is indifferent between reelecting and not reelecting a low-ability incumbent politician who maximized social welfare. In Prop. 3.1, we computed $E(u^E_{SL}|E_4)$ by assuming that the society did not reelect such a politician with probability one; now it may be useful to re-express it by assuming that the society reelects her with probability one. Hence,

$$E(u^E_{SL}|E_4) = \frac{\theta + q(1 + \theta_H + p) + (1 - q)(p(1 + \theta_L + 1) + (1 - p)(n_L(1 + \theta_L + 1) + (1 - n_L)(\theta_H + p))}{2}.$$

Let us write now $E(u^T_{SL}|a<b)$ in a way that is comparable term-by-term with $E(u^E_{SL}|E_4)$,

$$E(u^T_{SL}|a<b) = \frac{\theta + q(p + \theta_H + p + (1 - p)d) + (1 - q)(p(1 + \theta_L + 1) + (1 - p)(n_L(1 + \theta_H + d) + (1 - n_L)(\theta_L + d)))}{2}.$$

The assumptions $p > \frac{d}{1+e+q}$ and $e > d$ imply that $p > d$. Comparing term-by-term $E(u^E_{SL}|E_4)$ and $E(u^T_{SL}|a<b)$, from $1 > p + (1 - p)d, 1 + \theta_L > \theta_L + d$ and $\theta_H + p > \theta_H + d$, we have that $E(u^E_{SL}|E_4) > E(u^T_{SL}|a<b)$. Hence, $\theta_H - \theta_L < \min \left\{ \frac{p-1}{q}, \frac{p}{1-q} \right\} \implies E \succ T$.

Let now $\theta_H - \theta_L \in \left( \frac{p}{1-q}, \frac{1-p}{q} \right)$. Also in equilibrium $E_3$ the society is indifferent between reelecting and not reelecting a low-ability incumbent politician who maximized social welfare. If we assume that it reelects her with probability one, we have that

$$E(u^E_{SL}|E_3) = \frac{\theta + q(p + \theta_H + p) + (1 - q)(p(1 + \theta_L + 1) + (1 - p)(n_L(1 + \theta_H + 1) + (1 - n_L)(\theta_H + p)))}{2}.$$

After straightforward but tedious algebraic manipulations, taking into account that $n_L = \frac{p}{1-q}$, $\frac{1-p}{q}$, we have that $E(u^E_{SL}|E_3) > E(u^T_{SL}|a<b)$ if (and only if)

$$d < \frac{1 - q}{1 - p} \frac{p(1 - p) + q^2(\theta_H - \theta_L)^2}{p + q(\theta_H - \theta_L)} \overset{\text{def}}{=} \bar{d} \tag{A.13}$$

The proof that this always occurs works in two steps. The first step calls for assuming that $p > \bar{d}$ and showing that this can never occur when $\theta_H - \theta_L \in \left( \frac{p}{1-q}, \frac{1-p}{q} \right)$, with $p + q < 1$.

From Eq. A.13, letting $x = \theta_H - \theta_L$, it is easy to see that, if $p + 2q < 2 - \sqrt{\bar{d}}$, then $p < \bar{d}$ for all $x$, while if $p + 2q \geq 2 - \sqrt{\bar{d}}$, then $p > \bar{d}$ if (and only if) $x \in (x_1, x_2)$, where

$$x_{1,2} = \frac{p(1 - p) + \sqrt{p^2(1 - p)^2 - 4p(1 - p)(1 - q)(1 - q - p)}}{2q(1 - q)} \tag{A.14}$$

We are focusing on the case where $\theta_H - \theta_L \in \left( \frac{p}{1-q}, \frac{1-p}{q} \right)$, which calls for $p + q < 1$. Hence, a necessary condition for $p > \bar{d}$ is $x_2 > \frac{p}{1-q}$, with $p + 2q \geq 2 - \sqrt{\bar{d}}$.

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From Eq. A.14, \( x_2 > \frac{p}{1-q} \) if (and only if)
\[
\sqrt{p^2(1-p)^2 - 4p(1-p)(1-q)(1-p)} > p(2q + 1 + p)
\] (A.15)

If \( p + 2q \geq 2 - \sqrt{p} \), the right hand side of Eq. A.15 is positive. Multiplying both sides of Eq. A.15 by themselves and appropriately simplifying, we have that \( x_2 > \frac{p}{1-q} \) if (and only if) \( p + q > 1 \). Hence, \( \theta_H - \theta_L \in \left( \frac{p}{1-q}, \frac{1-p}{1-q} \right) \implies p \leq d \).

The second step is immediate. The assumptions \( p > \frac{d}{1-q} \) and \( e > d \) imply that \( p > d \). Moreover, we have seen above that \( d \geq p \). Hence, \( d > d \). Eq. A.13 is always satisfied, so \( \theta_H - \theta_L \in \left( \frac{p}{1-q}, \frac{1-p}{1-q} \right) \Rightarrow \mathcal{E} \succ T \).

Let now \( \theta_H - \theta_L \in \left( \frac{1-p}{q}, \frac{1-p}{1-q} \right) \). From
\[
E(u^E_S | E_2) - E(u^T_S | a < b) = \frac{q(1-q)(\theta_H - \theta_L) - (1-p)(d-q)}{2},
\]
\( \mathcal{E} \succ T \) if (and only if) \( \theta_H - \theta_L > \frac{1-p}{q} - \frac{d-q}{1-q} \). Of course, \( \theta_H - \theta_L > \frac{1-p}{q} \implies \theta_H - \theta_L > \frac{1-p}{q} - \frac{d-q}{1-q} \). Hence, \( \theta_H - \theta_L \in \left( \frac{1-p}{q}, \frac{1-p}{1-q} \right) \Rightarrow \mathcal{E} \succ T \).

Let finally \( \theta_H - \theta_L > \max \left\{ \frac{1-p}{q}, \frac{p}{1-q} \right\} \). From
\[
E(u^E_S | E_1) - E(u^T_S | a < b) = \frac{q(1-q)(\theta_H - \theta_L) - (1-p)d}{2},
\]
\( \mathcal{E} \succ T \) if (and only if) \( \theta_H - \theta_L > \frac{(1-p)d}{q(1-q)} \). Taking into that if \( p + q > 1 \), then \( \frac{p}{1-q} > \frac{(1-p)d}{q(1-q)} \), and that if \( p + q < 1 \), then \( \frac{1-p}{q} > \frac{(1-p)d}{q(1-q)} \), we have that \( \theta_H - \theta_L > \max \left\{ \frac{1-p}{q}, \frac{p}{1-q} \right\} \Rightarrow \mathcal{E} \succ T \). ■

**Proofs of Propositions 4.1 and 4.2**

The proofs of Prop. 4.1 and 4.2 work exactly as the proof of Prop. 3.1.

If in the second period the society reelects an incumbent politician who played \( W \), it will play \( F \), so the optimal values of \( s_{HW} \) and \( s_{LW} \) are still characterized by eqq. A.1 and A.3. On the contrary, if it reelects an incumbent politician who played \( R \), it will play \( T \). From
\[
E_{uS}(2, HR) - E_{uS}(2, new) = \theta_H + d - \hat{\theta} - p,
\]

\[
s_{HR} = \begin{cases} 1 & \text{if } \theta_H - \theta_L > \frac{p-d}{q} \\ 0 & \text{if } \theta_H - \theta_L < \frac{p-d}{q} \end{cases}
\] (A.16)

and from
\[
E_{uS}(2, LR) - E_{uS}(2, new) = \theta_L + d - \hat{\theta} - p < 0,
\]

\[
s_{LR} = 0
\] (A.17)

As for the optimal strategy for the non-congruent politician, from
\[
E_{uN}(W, s_{HW}) - E_{uN}(R, s_{HR}) = \frac{a + s_{HW}}{2} - \frac{1 + s_{HR}}{2},
\]

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taking into account that \( s_{HW} = 1 \) (Eq. A.1),

\[
n_H = \begin{cases} 
1 & \text{if } s_{HR} b < a \\
0 & \text{if } s_{HR} b > a \\
[0,1] & \text{if } s_{HR} b = a 
\end{cases} \tag{A.18}
\]

and from

\[
E_{NL}(W, s_{LW}) - E_{NL}(R, s_{LR}) = \frac{a + s_{LW}}{2} - \frac{1 + s_{LR} b}{2},
\]

taking into account that \( s_{LR} = 0 \) (Eq. A.17),

\[
n_L = \begin{cases} 
1 & \text{if } s_{LW} > 1 - a \\
0 & \text{if } s_{LW} < 1 - a \\
[0,1] & \text{if } s_{LW} = 1 - a 
\end{cases} \tag{A.19}
\]

The last step of the proof calls for considering a reelection strategy, determining the best response for the two types of non-congruent politicians and checking whether the assumed reelection strategy is actually optimal.

Let \( s_{HR} = 1 \) and \( s_{LW} = 0 \). From Eq. A.18, \( n_H = 1 \) if \( a > b \) and \( n_H = 0 \) if \( a < b \); from Eq. A.19, \( n_L = 0 \). Hence, from Eq. A.16, we actually have that \( s_{HR} = 1 \) if

\[
\theta_H - \theta_L > \frac{p - d}{1 - q} \tag{A.20}
\]

and, from Eq. A.3, \( s_{LW} = 0 \) if

\[
\theta_H - \theta_L \geq \frac{1 - p}{q}. \tag{A.21}
\]

If \( a > b \) and Eqs. A.20–A.21 are satisfied, there exists a perfect Bayesian equilibrium with \( s_{HW} = 1, s_{HR} = 1, s_{LW} = 0, s_{LR} = 0, n_H = 1 \) and \( n_L = 0 \) (equilibrium C1 in Prop. 4.1). The per-period expected utility for the society is

\[
E(u_{ES}^{C_1} | a > b) = \frac{\theta + p(1-p)q + q(\theta + p) + (1-q)(\theta + p)}{2}
\]

which can be rewritten as Eq. 4.1.1. On the other hand, if \( a < b \) and Eqs. A.20–A.21 are satisfied, there exists a perfect Bayesian equilibrium with \( s_{HW} = 1, s_{HR} = 1, s_{LW} = 0, s_{LR} = 0, n_H = 0 \) and \( n_L = 0 \) (equilibrium C1 in Prop. 4.2). The per-period expected utility for the society is

\[
E(u_{ES}^{C_1} | a < b) = \frac{\theta + p(1-p)(\theta + p) + (1-q)(\theta + p)}{2}
\]

which can be rewritten as Eq. 4.1.3.

Using exactly the same procedure for all the other reelection strategies \( s_{ij} \in [0,1], i \in \{H,L\}, j \in \{W,R\} \), with \( s_{HW} = 1 \) and \( s_{LR} = 0 \), one can easily find all the other equilibria. Equilibria C1–C4 represent therefore all the perfect Bayesian equilibria of the game. The sets of parameters that give rise to the different equilibria are disjoint, so for a given set of parameters the game has a unique perfect Bayesian equilibrium.
Proof of proposition 4.3

If \( \theta_H - \theta_L > \frac{1-p}{q} \), the society’s per-period expected utility under \( C \) is \( E(u^C_S | a>b, C_1=C_2) \). From \( E(u^C_S | a>b, C_1=C_2) > E(u^T_S | a>b) \) if (and only if) \( \theta_H - \theta_L > \frac{1-p}{q} \), we have that \( \theta_H - \theta_L > \frac{1-p}{q} \implies C > T \).

On the other hand, if \( \theta_H - \theta_L < \frac{1-p}{q} \), the society’s per-period expected utility under \( C \) is \( E(u^C_S | a>b, C_3=C_4) \). From \( E(u^C_S | a>b, C_3=C_4) > E(u^T_S | a>b) \) if (and only if) \( \theta_H - \theta_L > \frac{1-p}{q} \), we have that \( \theta_H - \theta_L < \frac{1-p}{q} \implies T > C \).

Proof of proposition 4.4

If \( \theta_H - \theta_L > \max \left\{ \frac{1-p}{q}, \frac{p-d}{1-q} \right\} \), the society’s per-period expected utility under \( C \) is \( E(u^C_S | a<b, C_1) \). From \( E(u^C_S | a<b, C_1) > E(u^T_S | a<b) \) if (and only if) \( \theta_H - \theta_L > \frac{1-p}{q} \), taking into account that \( d < 1 \), we have that \( \theta_H - \theta_L > \max \left\{ \frac{1-p}{q}, \frac{p-d}{1-q} \right\} \implies C > T \).

If \( \theta_H - \theta_L \in \left( \frac{p-d}{1-q}, \frac{1-p}{q} \right) \), the society’s per-period expected utility under \( C \) is \( E(u^C_S | a<b, C_2) \). We have shown above that, if \( \theta_H - \theta_L > \frac{1-p}{q} \), then \( E(u^C_S | a<b, C_1) > E(u^C_S | a<b) \). Taking into account that, for all \( \theta_H - \theta_L \), \( E(u^C_S | a<b, C_2) > E(u^C_S | a<b, C_1) \), we have that \( E(u^C_S | a<b, C_2) > E(u^T_S | a<b) \). Hence, \( \theta_H - \theta_L \in \left( \frac{p-d}{1-q}, \frac{1-p}{q} \right) \implies C > T \).

If \( \theta_H - \theta_L \in \left( \frac{p-d}{1-q}, \frac{1-p}{q} \right) \), the society’s per-period expected utility under \( C \) is \( E(u^C_S | a<b, C_3) \). In equilibrium, the society is indifferent between reelecting and not reelecting a low-ability incumbent politician who maximized social welfare. In prop. 4.2, we computed \( E(u^C_S | a<b, C_3) \) by assuming that the society did not reelect such a politician with probability one; now it may be useful to re-express it by assuming that the society reelects her with probability one. Hence,.

\[
E(u^C_S | a<b, C_3) = \theta + q(p+\theta_H+p+(1-p)d)+(1-q)(p(1+\theta_L+1)+(1-p)(n_L(1+\theta_L)+(1-n_L)(\theta_L+d)))
\]

Let us now write \( E(u^T_S | a<b) \) in a way that is comparable term-by-term with \( E(u^C_S | a<b, C_3) \).

\[
E(u^T_S | a<b) = \theta + q(p+\theta_H+p+(1-p)d)+(1-q)(p(1+\theta_L+1)+(1-p)(n_L(1+\theta_L)+(1-n_L)(\theta_L+d)))
\]

From \( 1+\theta_L > \theta_L + d \) and \( \theta + p > \theta_L + d \), we have that \( E(u^C_S | a<b, C_3) > E(u^T_S | a<b) \).

Hence, \( \theta_H - \theta_L \in \left( \frac{p-d}{1-q}, \frac{1-p}{q} \right) \implies C > T \).

Finally, if \( \theta_H - \theta_L < \min \left\{ \frac{1-p}{q}, \frac{p-d}{1-q} \right\} \), the society’s per-period expected utility under \( C \) is \( E(u^C_S | a<b, C_4) \). From \( E(u^C_S | a<b, C_4) > E(u^T_S | a<b) \), taking into account that, for all \( \theta_H - \theta_L \), \( E(u^C_S | a<b, C_4) > E(u^C_S | a<b, C_3) \), we have that \( E(u^C_S | a<b, C_4) > E(u^T_S | a<b) \). Hence, \( \theta_H - \theta_L < \min \left\{ \frac{1-p}{q}, \frac{p-d}{1-q} \right\} \implies C > T \).
Proof of proposition 4.5

Let firstly $\theta_H - \theta_L > \max \left\{ \frac{1-p}{q}, \frac{p}{1-q} \right\}$, so that equilibrium $E_1$ obtains under $E$, while equilibrium $C_1$ obtains under $C$. From $E(u_S^E | E_1) < E(u_S^C | a>b,C_1=C_2)$, we have that $\theta_H - \theta_L > \max \left\{ \frac{1-p}{q}, \frac{p}{1-q} \right\} \Rightarrow C \succ E$.

Let now $\theta_H - \theta_L \in \left( \frac{1-p}{q}, \frac{1-p}{1-q} \right)$, so that equilibrium $E_2$ obtains under $E$, while either equilibrium $C_1$ or equilibrium $C_2$ obtains under $C$. From $E(u_S^E | E_2) = E(u_S^C | a>b,C_1=C_2)$, we have that $\theta_H - \theta_L \in \left( \frac{1-p}{q}, \frac{p}{1-q} \right) \Rightarrow C \sim E$.

Let now $\theta_H - \theta_L \in \left( \frac{1-p}{q}, \frac{1-p}{1-q} \right)$, so that equilibrium $E_3$ obtains under $E$, while equilibrium $C_3$ obtains under $C$. From $E(u_S^E | E_3) < E(u_S^C | a>b,C_3=C_4)$, we have that $\theta_H - \theta_L \in \left( \frac{1-p}{q}, \frac{p}{1-q} \right) \Rightarrow C \succ E$.

Let finally $\theta_H - \theta_L < \min \left\{ \frac{1-p}{q}, \frac{p}{1-q} \right\}$, so that equilibrium $E_4$ obtains under $E$, while either equilibrium $C_3$ or equilibrium $C_4$ obtains under $C$. From $E(u_S^E | E_4) = E(u_S^C | a>b,C_3=C_4)$, we have that $\theta_H - \theta_L < \min \left\{ \frac{1-p}{q}, \frac{p}{1-q} \right\} \Rightarrow C \sim E$. $\blacksquare$

Proof of proposition 4.6

Let firstly $\theta_H - \theta_L > \max \left\{ \frac{1-p}{q}, \frac{p}{1-q} \right\}$, so that equilibrium $E_1$ obtains under $E$, while equilibrium $C_1$ obtains under $C$. From $E(u_S^E | E_1) < E(u_S^C | a>b,C_1)$, we have that $\theta_H - \theta_L > \max \left\{ \frac{1-p}{q}, \frac{p-d}{1-q} \right\} \Rightarrow C \succ E$.

Let now $\theta_H - \theta_L \in \left( \frac{1-p}{q}, \frac{1-p}{1-q} \right)$, so that equilibrium $E_2$ obtains under $E$, while under $C$ equilibrium $C_1$ obtains if $\theta_H - \theta_L > \max \left\{ \frac{1-p}{q}, \frac{p-d}{1-q} \right\}$, and equilibrium $C_2$ obtains otherwise. From $E(u_S^E | E_2) > E(u_S^C | a>b,C_1)$ and $E(u_S^E | E_2) = E(u_S^C | a<b,C_2)$, we have that $\theta_H - \theta_L \in \left( \max \left\{ \frac{1-p}{q}, \frac{p-d}{1-q} \right\}, \frac{1-p}{1-q} \right) \Rightarrow E \succ C$ and $\theta_H - \theta_L \in \left( \frac{1-p}{q}, \frac{p-d}{1-q} \right) \Rightarrow C \sim E$.

Let now $\theta_H - \theta_L \in \left( \frac{p-d}{1-q}, \frac{1-p}{1-q} \right)$, so that equilibrium $E_3$ obtains under $E$, while equilibrium $C_3$ obtains under $C$. From $E(u_S^E | E_3) < E(u_S^C | a>b,C_3)$, we have that $\theta_H - \theta_L \in \left( \frac{p-d}{1-q}, \frac{1-p}{1-q} \right) \Rightarrow C \succ E$.

Let finally $\theta_H - \theta_L < \min \left\{ \frac{p-d}{1-q}, \frac{1-p}{1-q} \right\}$, so that equilibrium $E_4$ obtains under $E$, while under $C$ equilibrium $C_3$ obtains if $\theta_H - \theta_L \in \left( \frac{p-d}{1-q}, \min \left\{ \frac{1-p}{q}, \frac{p-d}{1-q} \right\} \right)$, and equilibrium $C_4$ obtains otherwise. From $E(u_S^E | E_4) > E(u_S^C | a>b,C_3)$ and $E(u_S^E | E_4) = E(u_S^C | a>b,C_4)$, we have that $\theta_H - \theta_L \in \left( \frac{p-d}{1-q}, \min \left\{ \frac{1-p}{q}, \frac{p-d}{1-q} \right\} \right) \Rightarrow E \succ C$ and $\theta_H - \theta_L \in \left( 0, \min \left\{ \frac{1-p}{q}, \frac{p-d}{1-q} \right\} \right) \Rightarrow C \sim E$.

We have shown that $C \succ E$ if (and only if) either $\theta_H - \theta_L > \max \left\{ \frac{1-p}{q}, \frac{p}{1-q} \right\}$ or $\theta_H - \theta_L \in \left( \frac{p}{1-q}, \frac{1-p}{1-q} \right)$, i.e., if $\theta_H - \theta_L > \frac{p}{1-q}$, and that $C \sim E$ if (and only if) either $\theta_H - \theta_L \in \left( \frac{1-p}{q}, \frac{p-d}{1-q} \right)$ or $\theta_H - \theta_L \in \left( 0, \min \left\{ \frac{1-p}{q}, \frac{p-d}{1-q} \right\} \right)$, i.e., if $\theta_H - \theta_L < \frac{p-d}{1-q}$. $\blacksquare$