Complementary Assets, Start-Ups and Incentive to Innovate

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Abstract

In this paper we examine in a game theoretic framework in how far market conditions facilitating start-up formation positively affect technical change and firms' profits. We consider a model in which R&D efforts of an incumbent firm generate technological know-how embodied in key R&D employees, who might use this know-how to form a start-up. Market conditions, in particular the availability of complementary assets, influence whether new firms are created and determine expected profits for start-up-founders. Easy availability of complementary assets has the direct effect that the generation of start-ups, which leads to the diffusion and duplication of know-how, is fostered. However, incentives of incumbent firms to invest in R&D might be reduced because of the increased danger of knowledge loss through spin-out formation. We fully characterize the effects of an increase in the availability of complementary assets, demonstrating that under certain market conditions the effects on innovative activities and industry profits can be negative.

Keywords: Complementary Assets, Technical Change, R&D Effort, Startup

JEL classification: L20, M13, O30

1 Introduction

Innovation and diffusion of technology are widely recognized as being important drivers of economic growth. A large literature has evolved studying at different levels of aggregation the mechanisms by which innovative technologies come about and make their way into the economy. It is argued that the flows of knowledge embodied in employees often facilitate technological diffusion. A particular form of key employees' mobility is the creation of start-up firms. Empirical evidence shows indeed that the evolution of

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many industries is to a large extent driven by the entry of start-ups formed by former employees of incumbents (see, e.g., Dahl, Pedersen, and Dalum (2003); Klepper and Sleeper (2005); Klepper (2004)). Accordingly, start-up generation is seen as an important mechanism for the propagation of technical change and economic growth (see, e.g., Acs and Plummer (2005), Mueller (2007), or Audretsch, Keilbach, and Lehmann (2006)). However, as pointed out by Klepper (2001, p. 639) start-ups are sometimes seen as “parasites feeding off the innovative efforts of their parents, aided by ‘vulture’ capitalists that help them get started”, alluding to the possibility of key employees leaving their employment to start their own firm in the same industry. Arguably, the threat of the establishment of a start-up can have a distorting effect on the incentives of incumbents to innovate.

To better understand the tension between these two opposite forces one needs to further investigate the factors presiding start-up formation. Much of the literature has stressed that the skills and know-how acquired in previous employment are important factors for start-up formation (see, e.g., Gompers, Lerner, and Scharfstein (2005), and Cooper, Woo, and Dunkelberg (1989)). Furthermore, starting with Teece (1986) a rich literature has pointed out the importance of different kinds of complementary assets for the creation of new firms. These assets range from access to distribution and purchasing channels to organisational and managerial skills\(^1\).

Easy access to complementary assets, encouraging the formation of start-ups is generally regarded as a factor facilitating innovation and fostering technological change\(^2\). More complementary assets, however, by rendering easier for a key employee to found a start-up, imply also a potential knowledge loss and increased costs related to threatened start-up formation, adversely affecting incumbent firms’ incentives to invest in innovation. While the beneficial effects of an economic environment favorable to start-up formation are well understood, we still lack a clear understanding of its consequences on incumbent firms’ innovative efforts.

Our main goal in this paper is to shed light on the latter effects, by investigating the impact of the availability of complementary assets on the strategic interactions between an incumbent firm and a key R&D employee when property rights are not enforceable\(^3\). The firm is assumed to make R&D expenses to generate new knowledge, which is however embodied in the employee (a positive externality increasing her outside options). The latter faces the opportunity to found a start-up that will eventually compete with the incumbent in the same industry. In order to do so, however, she needs to have access to a key complementary asset. If the employee leaves the firm to found a start-up, only a fraction of the knowledge generated through R&D remains with the incumbent, who then suffers a reduction in profits both because of its inability to fully appropriate the returns from R&D investments and because of the presence of

\(^1\)Needless to say, many other variables have an influence on the start-up formation process. For instance, Hellmann (2007) shows that innovation and entrepreneurship are influenced by company policies as well as by the entrepreneurial environment and the allocation of intellectual property rights.

\(^2\)An example of complementary asset that has received substantial attention in the literature is the availability of financial resources and especially venture capital. For instance, the country reports of the European Innovation Scoreboard, published annually by the European Commission, regard early stage venture financing as a key indicator of the innovation potential of a region.

\(^3\)See for example Anton and Yao (1994, 1995) for a discussion of the many instances in which property rights are weak or absent.
a further competitor in the industry.

The value of the knowledge stock generated by R&D activities is common knowledge, so that the firm and the employee bargain over the size of the wage premium that the firm is willing to pay in order to prevent the employee from leaving. In case of disagreement, the employee leaves the firm (to found a start-up) and her payoff is then determined through bargaining with the provider of key complementary assets. Hence, the value of the employee's outside option depends ultimately on the availability of these assets.

On the one hand, by choosing the level of R&D effort, the firm influences the employee's incentives to create a start-up and therefore whether in equilibrium a new firm will form or not. On the other hand, R&D expenses positively affect the incumbent's profits for a given market structure. The interplay of these two effects determines the investment decision of the firm, crucially depending on the availability of complementary assets.

If complementary assets are easily available, the employee's outside option when negotiating employment conditions with the firm (given the employee opportunity of founding a start-up) is of high value. Potentially, this entails a higher probability of disagreement between the firm and the key employee (resulting in the formation of a start-up), or a larger compensation received by the employee in case an agreement is reached. Both effects imply reduced incentives for the incumbent to invest in the accumulation of R&D knowledge. Once a start-up is formed, however, the overall bargaining power of the employee when dealing with the provider of the complementary assets only affects the allocation of the start-up profits between the two, without influencing any longer the firm's innovative efforts. Hence, the incumbent's investment in R&D knowledge is strictly decreasing for levels of complementary assets availability at which a start-up does not form, and remains constant, although inefficiently provided, for higher degrees of availability. Two sources of inefficiencies arise: ex ante, there is under-investment associated to the distortions in the incumbent's incentives to innovate, and ex post, there is the possibility that no start-up is formed even if aggregate industry profits would rise should start-up formation occur.

It is interesting to note that, although an increase in the availability of complementary assets harms the incumbent by improving the outside options of the key employee, the latter does not necessarily benefit from this increase. There are indeed two opposite effects at work: a direct effect, implying that for a given level of R&D knowledge an increase in the employee's outside options rises the fraction of the incumbent's profits she obtains; and a strategic effect, due to the decreasing incentives of the firm to invest in R&D, which in turn reduces firm profits. Taken together the direct and the strategic effects imply that the employee receives a higher fraction of a shrinking joint surplus, so that the larger availability of complementary assets can indeed harm the employee as well as the incumbent. In fact, we will show that there are cases in which ex post, given the R&D investment of the incumbent, the employee has an incentive to create a start-up, although she would be better off committing ex ante not to form it. A result that resembles closely to a hold-up problem.

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4 More generally, it is clear that the parent firm's investment decisions depend on its overall bargaining power when dealing with the potential start-up founder. Besides the availability of complementary assets, other factors affect start-up formation, such as the institutional setup and market characteristics.
The main argument sketched above has important policy implications. It is commonly agreed that economic environments facilitating start-up formation support the diffusion of new ideas and innovations and, via this channel, economic growth (see e.g. Acs and Szerb (2006)). On the contrary, we argue that such economic environments may have a negative distorting impact on firms’ R&D expenditures. Even in the limit case in which all necessary complementary assets become “freely” available (e.g. because of policy programs), the incumbent still under-invests in R&D due to the effect of complementary assets availability on its incentives, which may cast doubts also on the implications of public programs aimed at providing support to potential start-ups.

Investigating the tradeoff between positive (reduction of ex-post inefficiencies) and negative (increase of investment distortions) effects of economic environments that foster start-up formation is the key contribution of this paper. To put our results in the right perspective, however, it is important to stress that several positive effects of start-up formation commonly emphasized in the literature (such as knowledge spillovers, inter-temporal effects of cluster formation; see, e.g., Audretsch and Feldman (2004)) are not taken into account here.

Two streams of literature are closely related to our analysis. First, several papers have focused on the effects of potential worker mobility on investment and technology choice decisions of firms. For instance, Gersbach and Schmutzler (2003) observe that the possibility of employees moving to direct competitors has a negative impact on the equilibrium investments in innovative activities. Böhm and Colombo (2006) show in a general equilibrium framework that the possibility of experienced employees becoming entrepreneurs may discourage the choice to adopt better technologies by incumbent firms. Although both papers consider spillovers through the labor market, the effects of complementary assets on start-up formation do not play any role there. The negative strategic effect arising in our framework resembles also the impact of weak patent systems on incentives to invest in innovative activities, which has been extensively discussed in the literature (see, e.g., the survey by Gallini and Scotchmer (2002)). Second, the factors influencing if and how start-ups are formed have been investigated in a number of papers. Related to our setup are, in particular, Anton and Yao (1994, 2002), Anand and Galetovic (2000, 2004) and Baccara and Razin (2004), who study strategies to prevent, or form and finance, the formation of start-ups by employees of an incumbent firm. While these papers carefully analyze different aspects of the start-up formation process with absent or weak property rights, they do not link these considerations to the ex-ante incentives of firms to invest in innovative activities.

The paper is organized as follows. Section 2 describes the model. Section 3 investigates the influence of complementary assets on the investment in innovation by the incumbent firm, and the process of start-up formation. Section 4 studies the impact of the allocation of bargaining power between the firm and the employee on start-up formation, while Section 5 concentrates on the possibility of the formation of a spinoff where the complementary assets are provided by the incumbent. Finally, Section 6 focuses on policy implications, and on a discussion of the main results. In Appendix A we exemplify our general framework through a standard duopoly model with linear demand and quality improving R&D. All figures in the paper have been produced using this example. Appendix B contains all proofs and technical details.
2 The Model

We consider the interaction between a firm (F) and a key R&D employee (E). The firm invests $I(K_E)$, in order to generate new knowledge $K_E$, which is however embodied in the employee. The employee may found a start-up company, which then competes with firm F. In order to do so complementary assets are needed, which may be financial resources, organizational and managerial know-how or the access to relevant factor markets and distribution channels. The assets are provided by a third party at a cost $M_s(K_E)$. The more advanced the technology is the larger are the costs of providing these assets. If no start-up is created the firm earns a profit $\pi_{mF}(K_E)$ in the market. In case E establishes a start-up she takes her embodied knowledge ($K_E$) with her leaving a knowledge stock $K_F \leq K_E$ with the firm. The profits earned by the incumbent firm and the employee (founding a start-up) in this case are $\pi_{sF}(K_F, K_E)$ and $\pi_{sE}(K_F, K_E)$, respectively. The strategic incentives of the firm and of the employee are analyzed in a two-stage game (see Figure 1):

**Stage 1:** the firm chooses the stock of knowledge $K_E$ to be generated, and sustains the costs $I(K_E)$. The knowledge is embodied in the employee. If she leaves, only a stock $K_F = \delta K_E, 0 < \delta < 1$, remains in the firm.

We make standard assumptions concerning decreasing marginal effects of R&D efforts. Given that if a start-up forms the knowledge stocks of both the incumbent firm and the start-up are functions of $K_E$, in what follows we simplify the notation by writing $\pi_{mF}(K_E), \pi_{mE}(K_E)$ instead of $\pi_{sF}(\delta K_E, K_E), \pi_{sE}(\delta K_E, K_E)$.
Assumption 1
\[ \pi(0) = 0, \pi(K_E)' > 0 \]
for \( \pi = \pi_m, \pi_s, \pi_E \). Furthermore,
\[ M_s(0) = 0, M_s(K_E)' > 0. \]

Define \( \Delta \pi_F \) as the loss in profits of the incumbent firm when a start-up is formed and \( \Delta \pi_E \) as the net profit of the start-up.
\[ \Delta \pi_F(K_E) := \pi_m(K_E) - \pi_s(K_E), \]
\[ \Delta \pi_E(K_E) := \pi_s(K_E) - M_s(K_E). \]

We assume that the creation of a start-up harms the incumbent firm and that the marginal return on additional knowledge for the incumbent is always positive and larger if the employee stays in the firm than if she creates a start-up. Also, since our focus is on the impact of the threat of start-up formation on investment incentives of the incumbent firm, we let the net profits of a start-up be positive and larger than the losses suffered by the incumbent. Hence, it would be efficient to form a start-up. Moreover, this effect becomes stronger the larger the knowledge stock of the employee is. Furthermore, we assume that higher knowledge implies a larger market size, meaning that the positive effect of knowledge duplication on total industry profits becomes larger the more is invested in R&D. This is summarized by:

Assumption 2
\[ \Delta \pi_E(K_E) > \Delta \pi_F(K_E) > 0, \]
\[ \Delta \pi'_E(K_E) > \Delta \pi'_F(K_E) > 0 \forall K_E > 0. \]

Our final assumption ensures that the optimal investment level is positive and finite.

Assumption 3 The shape of the cost function for generating knowledge guarantees positive optimal investment and convexity of net industry profits with respect to \( K_E \):
\[ I'(K_E) > 0, I'(0) < \pi_s'(0), \]
\[ I''(K_E) \geq \max \left\{ \frac{\partial^2 (\pi_F(K_E) + \pi_s(K_E) - M_s(K_E))}{\partial K_E^2}, \frac{\partial^2 \pi_m(K_E)}{\partial K_E^2} \right\}. \]

Stage 2: In the second stage the size of the knowledge stock \( K_E \) is common knowledge. The firm and the employee bargain over the size of a wage bonus \( b \) that the firm is willing to pay to prevent the employee from leaving. The bargaining power of the firm in these negotiations is denoted by \( \xi \in (0, 1) \). With probability \( \xi \) [subgame 2] the firm offers a bonus \( b_F \), which the employee accepts or rejects; with probability \( (1 - \xi) \) [subgame 3] the employee demands a bonus \( b_E \) and the firm accepts or rejects it. In
both cases, if the offer/demand is accepted, the employee stays in the firm and the payoffs of the two players are

$$P_{21}^F = \pi^m_F(K_E) - I(K_E) - b_F, \quad P_{21}^E = b_F, \quad P_{31}^F = \pi^m_F(K_E) - I(K_E) - b_E, \quad P_{31}^E = b_E.$$

If there is disagreement the employee leaves the firm and her payoff is determined through bargaining with a provider of the needed complementary assets. If the (former) employee is not able to obtain the necessary complementary assets from the current negotiation partner she has to find another provider. This would need search time, and therefore the payoffs earned by the employee after the search would have to be discounted by a factor $\alpha \in (0, 1)$. The more easily complementary assets are available the shorter is the necessary search time and the larger is $\alpha$. We assume that the outcome of the bargaining, which determines the distribution of the start-up ownership between the founder and the provider of the necessary assets, is given by a generalized Nash bargaining solution, where $\beta \in (0, 1)$ denotes the bargaining power of the employee. We denote by $x$ the amount allocated to the employee in the bargaining and assume for simplicity that all potential alternative providers are identical. Thus the threat-points of the start-up founder and the provider are $\alpha x$ and $M_s$, respectively. The joint surplus to be allocated reads

$$\pi^s_E(K_E) - M_s.$$

Accordingly, we get

$$x = \alpha x + \beta(\pi^s_E(K_E) - M_s - \alpha x),$$

and hence

$$x = \frac{\beta}{1 - \alpha(1 - \beta)}(\pi^s_E(K_E) - M_s).$$

Given that the employee has to search for a provider when deciding to found a start-up she expects the payoff

$$\alpha x = \gamma(\pi^s_E(K_E) - M_s),$$

where

$$\gamma = \frac{\alpha \beta}{1 - \alpha(1 - \beta)} \in (0, 1)$$

measures the overall bargaining power of the start-up founder when dealing with a provider of complementary assets. All-together, we obtain the following payoffs of the two players in this subgame:

$$P_{22}^F = P_{32}^F = \pi^s_E(K_E) - I(K_E)$$

$$P_{22}^E = P_{32}^E = \gamma \Delta \pi_E(K_E).$$

The parameter $\gamma$ is a natural proxy for the availability of complementary assets. Although the notion of $\gamma$ is rather abstract in our setup one can provide several economic interpretations of it. There are numerous reasons why the access of potential founders to assets needed for start-up formation may be restricted. For instance, private equity markets may not be sufficiently developed, necessary organizational, legal or managerial advice may be costly to access, market regulations may impose high barriers for

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establishment of a firm, or dominant incumbent firms may restrict the access of start-ups to vertically related ‘assets’ like distribution channels or the supply of production factors.

3 R&D Investment and Start-up Formation

The main goal of our analysis is to investigate the strategic effect of the availability of complementary assets on firm investment and start-up formation. The presence of a provider of complementary assets – by allowing the creation of a start-up – implies that the incumbent firm on the margin acquires less than the full return on investment. Accordingly, one would expect a downward distortion in the investment incentives of the incumbent.

Note that, by Assumption 2, the total market profits (those of the incumbent plus those of the potential start-up) are always larger when a start-up forms. Accordingly, the creation of a new firm is the efficient outcome, and the efficient investment level is given by

$$K_E^{\text{eff}} = \arg \max_{K_E} \left[ \pi_F^*(K_E) + \pi_E^*(K_E) - I(K_E) \right].$$

(3)

This efficient level is used as a benchmark in the following analysis of equilibrium outcomes.

The game described in the previous section is an extensive form game with perfect information. We characterize its subgame perfect Nash equilibria (SPNE) using backward induction. Considering subgame 2 in Figure 1 it is easy to see that whenever

$$P_{F21}^1 + P_{E21}^1 < P_{F22}^1 + P_{E22}^1$$

(4)

then in equilibrium a start-up is formed. Inequality (4) is equivalent to $G(K_E; \gamma) > 0$, where

$$G(K_E; \gamma) := -\Delta \pi_F(K_E) + \gamma \Delta \pi_E(K_E).$$

(5)

Notice that the same condition implies that a start-up emerges in equilibrium in subgame 3 as well. The function $G(K_E; \gamma)$ gives the joint net profits increase for the incumbent firm and the start-up. In equilibrium the employee creates a new firm if and only if the joint profit increase $G(K_E; \gamma)$ is positive.

We define a set $\hat{K}_E$ such that for all $K_E \in \hat{K}_E$ one has $G(K_E; \gamma) \leq 0$, meaning that a start-up does not form. If $K_E \in \hat{K}_E$ then in equilibrium the equalities

$$b_F(K_E) = \gamma \Delta \pi_E(K_E) > 0$$

and

$$b_E = \Delta \pi_F(K_E) > 0$$

hold.

Accordingly, at stage 1 the incumbent firm’s payoffs are

$$p_F^1(K_E; \gamma) = \begin{cases} H_m(K_E; \gamma) & \text{if } K_E \in \hat{K}_E \\ H_s(K_E) & \text{otherwise} \end{cases}$$

We explicitly indicate the dependence of $G$ on $\gamma$ since our analysis focuses on variations in $\gamma$. **
where

$$H_m(K_E; \gamma) := \pi_p(K_E) - \xi G(K_E; \gamma) - I(K_E)$$  \hspace{1cm} (6)

$$H_s(K_E) := \pi_p(K_E) - I(K_E).$$  \hspace{1cm} (7)

Under Assumption (3), $H_m(K_E; \gamma)$ and $H_s(K_E; \gamma)$ are strictly concave functions. Note further that $K_E \in \hat{K}_E$ if and only if $H_m(K_E; \gamma) \geq H_s(K_E; \gamma)$. Therefore, at stage 1, the firm maximizes over the upper envelope of $H_m$ and $H_s$.

We define:

$$\bar{K}_{E,m}(\gamma) = \arg\max_{K_E} H_m(K_E; \gamma) > 0, \quad \bar{K}_{E,s} = \arg\max_{K_E} H_s(K_E) > 0,$$  \hspace{1cm} (8)

where the positivity of the argmax follows from Assumption 3. It should be noted that $H_s(K_E)$ and hence also $\bar{K}_{E,s}$ are independent of $\gamma$. Define

$$K_E^*(\gamma) = \arg\max_{K_E} p_F(K_E; \gamma)$$

as the equilibrium level of the incumbent’s R&D investment. Clearly, one has that

$$K_E^*(\gamma) \in \{\bar{K}_{E,m}(\gamma), \bar{K}_{E,s}\}.$$

It should also be noted that, on the one hand, by choosing $K_E$ the firm influences the employee’s incentives to create a new firm, and therefore whether in equilibrium a start-up forms or not. On the other hand, $K_E$ affects the incumbent’s profits for a given market structure. The interplay of these two effects determines the optimal investment, which crucially depends on the bargaining power of the employee with the provider of the complementary assets, $\gamma$. As for the impact of $\gamma$ on $\bar{K}_{E,m}(\gamma)$ and $\bar{K}_{E,s}$, we obtain

**Lemma 1**

(i) $\frac{\partial H_m(K_E; \gamma)}{\partial \gamma} < 0 \forall K_E$

(ii) $\frac{\partial \bar{K}_{E,m}(\gamma)}{\partial \gamma} < 0$; $\frac{\partial \bar{K}_{E,s}}{\partial \gamma} = 0$

(iii) There exists a unique $\hat{\gamma} \in (0,1)$ such that $\bar{K}_{E,m}(\hat{\gamma}) = \bar{K}_{E,s}$.

(iv) $\frac{\partial \hat{\gamma}}{\partial \xi} = 0$

Points ii) and iii) of the Lemma are illustrated in Figure 2 where we depict the typical shapes of the investment levels $\bar{K}_{E,m}(\gamma)$ and $\bar{K}_{E,s}$ as functions of $\gamma$. 6

It is now straightforward to characterize the SPNE of the game for different values of $\gamma$.

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6 All figures are drawn for a particular instance of our setting with a linear demand structure and vertical and horizontal differentiation between the goods offered by the incumbent and the start-up. The corresponding model, based on Symeonidis (2003), is described in detail in Appendix A.
Figure 2: Investment levels $\bar{K}_{E,m}(\gamma)$ and $\bar{K}_{E,s}$ for $\gamma \in [0,1]$

Proposition 2

(i) There exists a unique value $\bar{\gamma} \in (0,1]$ such that, for all $\gamma < \bar{\gamma}$, in the unique SPNE one has $K_E^*(\gamma) = \bar{K}_{E,m}$ and no start-up is formed. For all $\gamma > \bar{\gamma}$, in the unique SPNE one has $K_E^*(\gamma) = \bar{K}_{E,s}$ and a start-up is formed.

(ii) For all $\gamma$, $K_E^*(\gamma) < K_{E,eff}^*$. 

The proposition illustrates the strategic effects of the availability of complementary assets on the firm’s R&D effort. If complementary assets are easily available (high $\gamma$), then the employee’s outside option when negotiating employment conditions with the firm (given the employee’s opportunity of founding a start-up) is high. This has three potential effects: first, it increases the possibility of disagreement, which results in the establishment of a start-up; second, in the case of agreement, it increases the compensation that the employee receives from the firm; and third, it reduces the firm’s incentives to invest in the generation of new knowledge.

The last effect is formally illustrated by the observation that $\bar{K}_{E,m}(\gamma)$ is decreasing in $\gamma$. Note, however, that when a start-up is created, then $\gamma$ only influences the net surplus allocation between the employee and the provider of complementary assets; hence, the incumbent’s incentives to invest in R&D are independent of $\gamma$ (i.e. $\bar{K}_{E,s}$ is constant in $\gamma$). The second part of the proposition shows that, no matter whether a start-up is formed or not, the firm’s incentives to invest fall short of the efficient benchmark due to the fact that part of the total surplus generated by start-up formation are not appropriated by the firm.

We now investigate how the incentives to invest in R&D in the start-up case compare with those in the case in which the firm retains the employee. Intuition might suggest
that the firm’s incentives to invest are larger if in equilibrium no start-up is formed and all the knowledge generated stays within the incumbent firm. In what follows, we will characterize under which circumstances the inequality $K_{E,m}(\gamma) > K_{E,s}$ is indeed satisfied. In order to do so we distinguish between two cases. Denote by $\epsilon_F(K_E)$ the elasticity of the incumbent’s losses due to start-up formation with respect to $K_E$, and by $\epsilon_E(K_E)$ the elasticity of the start-up’s profits with respect to $K_E$, i.e.:

$$
\epsilon_F(K_E) = \frac{\Delta\pi_F'(K_E) K_E}{\Delta\pi_F(K_E)}, \quad \epsilon_E(K_E) = \frac{\Delta\pi_E'(K_E) K_E}{\Delta\pi_E(K_E)}.
$$

If the elasticity of the incumbent’s losses caused by start-up formation is smaller than the elasticity of the start-up’s profits with respect to $K_E$, i.e. $\epsilon_F(K_E) < \epsilon_E(K_E)$ for all $K_E > 0$, we say that knowledge has a slow market expanding effect. If this inequality is reversed for all $K_E > 0$ we say that knowledge has a fast market expanding effect. Note that, on the one hand, in the case of a fast market expanding effect, for large levels of $K_E$, R&D investments have mainly a business stealing effect, meaning that most additional profits gained by the start-up are at the expenses of the incumbent firm. On the other hand, with slow market expansion, the positive effect of additional R&D expenditures on total industry profits grows with the total level of R&D effort. Figure 3 illustrates slow and fast market expanding effects for the linear demand case with quality improving R&D described in Appendix A.

Figure 3: Effect of R&D investments on $\Delta\pi_F$ and $\Delta\pi_E$ in case of slow market expanding (a) and fast market expanding (b) effects.

The following proposition shows that, on the one hand, when knowledge has a slow market expanding effect the availability of complementary assets must be high in order to foster the creation of a new firm. Furthermore, the optimal investment level of a firm that is able to prevent the employee from leaving is not necessarily above the level that would be optimal if a start-up forms. On the other hand, when knowledge has slow market expanding effect, whereas low elasticity implies a fast market expanding effect. See Appendix A for details.

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7In our example with quality improving R&D a crucial factor distinguishing between the cases of slow and fast market expanding effects is the elasticity of product quality with respect to R&D investments. A high elasticity corresponds to a slow market expanding effect, whereas low elasticity implies a fast market expanding effect. See Appendix A for details.
a fast market expanding effect, the minimal availability of complementary assets that leads to the creation of a new firm is comparatively small, and investments in cases where a start-up is founded are always smaller than in cases where the employee stays in the firm.

**Proposition 3**

(i) If knowledge has a slow market expanding effect, then $\gamma > \hat{\gamma}$ and for all $\gamma \in (\hat{\gamma}, \bar{\gamma})$ no start-up is created but the R&D investment of the incumbent is below the optimal investment under start-up formation: $\tilde{K}_{E,m}(\gamma) < \tilde{K}_{E,s}$.

(ii) If knowledge has a fast market expanding effect, then $\bar{\gamma} < \hat{\gamma}$. Whenever a start-up is not formed the R&D investment of the incumbent is above the optimal investment under start-up formation: $\tilde{K}_{E,m}(\gamma) > \tilde{K}_{E,s}$ for all $\gamma \leq \bar{\gamma}$.

Figure 4: Optimal R&D investment for varying availability of complementary assets ($\gamma$) in the cases of slow market expanding (a) and fast market expanding (b) effects of investment.

Figure 4 illustrates Proposition 3. The upward jump depicted in Figure 4(a) might be surprising at first, considering that for $\gamma < \hat{\gamma}$ the incumbent retains all generated knowledge in the firm, while for $\gamma > \hat{\gamma}$ it partly loses it. To gain an intuition for this observation, note that start-up formation occurs only if the induced increase in industry profits is large enough to account for the profit share accruing to the provider of the complementary assets. In the case of slow market expansion, the additional industry profit at the optimal investment level is (relatively) small. Hence, for a large range of values of $\gamma$, additional profits can not account for the profit share of the provider of the complementary assets, and no start-up is formed. This implies that the main effect of an increase of $\gamma$ is an improvement of the employee’s position in the bargaining process with the incumbent. Such a negative incentive effect occurs for a wider interval of $\gamma$ values than in the fast market expanding case. In fact, under slow market expansion, $\bar{\gamma}$ is so large that (at $\bar{\gamma}$) the negative incentive effect outweighs the direct effect of potential knowledge loss. Hence, anticipating the creation of a start-up increases the incumbent’s
incentives to invest in R&D. The opposite occurs in the fast market expanding case, in which (since $\bar{\gamma}$ is small) the range of $\gamma$ values such that no start-up forms is so small that the direct effect dominates.

The following corollary, directly implied by Lemma 1 and Proposition 3, summarizes our insights concerning the impact of the availability of complementary assets on the incumbent’s investment incentives.

**Corollary 4** If knowledge has slow (fast) market expanding effects, the optimal investment $K^*_E(\gamma)$ is strictly decreasing for $\gamma < \bar{\gamma}$, it exhibits an upward (downward) jump at $\gamma = \bar{\gamma}$ and it is constant for $\gamma > \bar{\gamma}$.

Also observe that Corollary 4 and Lemma 1 together imply that $K^*_E(\gamma) < \bar{K}_{E,m}(0)$ for all $\gamma \in (0, 1]$, where $\bar{K}_{E,m}(0)$ is the efficient level of R&D investments for all levels of $\gamma$ for which no start-up is formed. Moreover, it follows from Assumption 2 that the efficient level of $K_E$ is even larger than $\bar{K}_{E,m}(0)$ when a start-up is formed. Therefore, there is always under-provision of R&D effort by the incumbent, regardless of whether a start-up is generated or not.

We now turn to the dependence of profits on the availability of complementary assets. Denote by $P^*_F(\gamma)$ and $P^*_E(\gamma)$ the set of SPNE payoffs of the firm and the employee, respectively. In general, $P^*_F(\gamma)$ and $P^*_E(\gamma)$ are correspondences, but Lemma 9 in Appendix B establishes that these correspondences are almost everywhere single-valued and continuous. Concerning the behavior of the profits of the incumbent firm and of the start-up founder with respect to the availability of complementary assets, we can state:

**Proposition 5**

(i) The equilibrium profits of the incumbent firm are strictly decreasing in $\gamma$ for $\gamma < \bar{\gamma}$ and constant in $\gamma$ otherwise;

(ii) The equilibrium profits of the employee are strictly increasing in $\gamma$ for the range of $\gamma$-values where a start-up is formed;

(iii) If knowledge has a fast market expanding effect then the employee’s equilibrium profits exhibit a downward jump for increasing $\gamma$ at $\gamma = \bar{\gamma}$.

There is a clear economic intuition for Proposition 5. When a start-up is formed, the profits and the investment level of the incumbent are unaffected by $\gamma$, since this parameter does not determine the size of the start-up’s profits, but only their allocation between the provider of complementary assets and its founder. Given that the incumbent’s investment level does not change, the profits of the start-up founder are obviously increasing as her bargaining power with the provider of the required complementary assets increases. On the contrary, if no start-up is created, $\gamma$ affects the value of the employee’s outside options when bargaining with the firm, and the incumbent’s profits are strictly decreasing in $\gamma$. The fact that the payoff of the firm is decreasing in $\gamma$ does not necessarily imply that the employee profits from an increase in the availability of complementary assets. Two effects are at work: (i) a direct effect, implying that for a given level of $K_E$ an increase in the employee’s outside option raises the fraction of
the firm’s profits $\pi_m(K_E)$ she obtains; (ii) an indirect strategic effect, due to the fact that as $\gamma$ increases the incentives of the firm to invest in R&D, and accordingly its profits $\pi_m(K_E)$, diminish. Together, the direct and the strategic effects imply that the employee receives a larger fraction of a shrinking joint surplus, so that the net effect can be negative and a larger availability of complementary assets can harm the employee. Although for low and high values of $\gamma$ the employee’s payoff rises if the availability of complementary assets increases, there is an intermediate range in which increasing $\gamma$ has a negative effect on $P_E^*$. Since the incumbent’s profits are strictly decreasing in $\gamma$, in that range reducing the availability of complementary assets would induce a Pareto improvement. This effect is illustrated in Figure 5, from which it is also evident that for a slow market expanding effect the increase of $\gamma$ across the threshold $\bar{\gamma}$ might induce an upward jump of the employee’s profits. Contrary to the case of a fast market expanding effect, however, the direction of the jump in profits can not be determined in general.

Figure 5: Firm’s and employee’s profits for varying availability of complementary assets ($\gamma$) in the cases of slow market expanding effects of investment.

When knowledge has a fast market expanding effect the relevance of the strategic effect becomes particularly apparent if the parameter $\gamma$ moves from the range in which no start-up is formed to the start-up region. In this case, the strategic effect induces a downward jump in the firm’s R&D effort and, as shown in the third claim of Proposition 5, this results in a discontinuous reduction in the employee’s payoffs. *Ex post*, given the investment in R&D knowledge by the incumbent firm, the employee has an incentive to establish her own firm. However, she would be better off by committing *ex ante* not to form the start-up. Put differently a hold-up problem arises (inducing inefficient investment by the incumbent), which could be avoided if the employee were able to
commit not to leave the firm.

4 The Influence of the Incumbent’s Bargaining Power on Start-up Formation

The incumbent firm’s investment incentives are not only influenced by the availability of complementary assets but also by its bargaining power when negotiating with the potential start-up founder. We do not focus on the factors determining the firm’s bargaining power $\xi$, but we interpret it as a proxy of the intensity of competition in the labor market for workers with R&D skills. On the one hand, if the supply of potential R&D employees on the market is strong, the firm has a relatively strong position when bargaining with the potential start-up founder, and $\xi$ is large. On the other hand, if R&D skills are scarce on the market, $\xi$ is assumed to be low. It is straightforward to see that an increase in the incumbent’s bargaining power increases the minimal availability of complementary assets that leads to start-up formation.

**Lemma 6** The threshold $\bar{\gamma}$ at which a start-up forms is strictly increasing in the firm’s bargaining power $\xi$ whenever $\bar{\gamma} \neq \hat{\gamma}$. When $\bar{\gamma} = \hat{\gamma}$ for some $\xi \in (0, 1)$ then $\bar{\gamma} = \hat{\gamma}$ for all $\xi \in (0, 1)$.

An implication of Lemma 6 is that if the availability of complementary assets is too low for the formation of a start-up, but just slightly below the threshold $\bar{\gamma}$, then a decrease in the firm’s bargaining power with employee leads to the creation of a start-up. In the light of the interpretation of $\xi$ given above this implies that a decrease of the supply of potential R&D employees in the labor market can facilitate the creation of start-ups.

In the following proposition we show, on the one hand, that no start-ups are formed regardless of the firm’s bargaining power if the availability of complementary assets is low. On the other hand, when the availability of complementary assets is above a threshold, start-ups always form if the bargaining power of the firm is sufficiently small.

**Proposition 7** There exists a $\tilde{\gamma} \in [0, 1]$ such that if $\gamma < \tilde{\gamma}$ no start-up is formed regardless of the value of $\xi$. If $\gamma > \tilde{\gamma}$ then there exists a $\tilde{\xi} \in [0, 1]$ such that a start-up is formed if and only if $\xi \leq \tilde{\xi}$.

To gain an intuitive understanding of the results in Proposition 7, consider a situation in which the availability of complementary assets is just sufficient to trigger the creation of a start-up. An increase in the firm’s bargaining power $\xi$ does not affect its payoff as long as the formation of a start-up still occurs. However, the increase in $\xi$ induces an increase in the firm’s payoff should it decide to retain the employee. As a consequence, the firm’s profits might become larger under the employee’s retention than under the start-up formation scenario, in which case the increase in $\xi$ results in a discontinuous change in investment.

At first sight it might seem counterintuitive that a decrease in the employee’s bargaining power with the firm makes it more likely that she stays in the firm and no start-up is created. The driving force of this effect is the change in investment levels
induced by the increase in $\xi$. A higher investment level makes it more likely that start-up formation is efficient, and therefore facilitates it. At the same time, however, since the incumbent’s payoff is not affected by $\xi$ if a start-up forms, while it is increasing in $\xi$ if no start-up is created, the firm has an incentive to retain the employee for $\xi$ sufficiently large, and chooses its investment level accordingly.

5 Start-ups as Spinoffs of the Incumbent

In the analysis developed so far, three different parties are involved in the start-up formation process: the incumbent firm, the key employee and an external provider of complementary assets. The latter is needed to realize the potential benefits associated to the formation of the start-up, but his presence negatively influences the results of the bargaining between the incumbent and the employee. One may therefore ask whether the efficiency of the start-up formation process could be improved if no third party were involved. In order to address this issue, we consider a situation in which the incumbent is able and willing to directly provide the needed complementary assets to the employee. The resulting start-up is therefore a spinoff of the incumbent firm. In this case, the employee has the option to bargain with the incumbent the conditions of spinoff formation, instead of having to turn necessarily to an external provider of complementary assets. For analytical simplicity, we assume that the costs of providing the required complementary assets for the firm are identical to those of an external provider, and hence given by $M_s$. To incorporate the option that the firm can enable a spinoff, we alter the bargaining game between the firm and the employee in such a way that the agreement between the two can lead either to no spinoff formation with a bonus to the employee, or to the generation of a firm-enabled spinoff where the employee obtains a certain fraction of the profits. Which of these two options is chosen depends on the relative size of the joint surplus that is generated. As before, the bargaining power of the firm is denoted by $\xi$.

Formally, we have a bargaining game between the firm and the employee in which the outside options of the firm and of the employee are $\pi_F^m(K_E)$ and $\gamma \Delta \pi_E(K_E)$, respectively. According to our considerations above the joint surplus is given by

$$S = \max \left[ \pi_F^m(K_E), \pi_F^s(K_E) + \Delta \pi_E(K_E) \right].$$

Since total industry profits are larger if a start-up (spinoff) is formed by Assumption 2, the maximum in the expression above is always attained for the case of spinoff formation, so that

$$S = \pi_F^s(K_E) + \Delta \pi_E(K_E).$$

It is easy to see that $S$ always exceeds the sum of the outside options. Hence, a spinoff forms, and the amount allocated to the firm in the Nash bargaining solution is given by $\pi_F^s(K_E) + \xi(1-\gamma)\Delta \pi_E(K_E)$. From this we obtain that the objective function of the firm when deciding about its R&D effort level is given by $\bar{H}_s(K_E, \gamma)$, with

$$\bar{H}_s(K_E, \gamma) = \pi_F^s(K_E) + \xi(1-\gamma)(\pi_E^s(K_E) - M_s) - I(K_E).$$
Accordingly, the optimal investment level of the firm in the case in which it has the resources to support a spinoff reads

$$\hat{K}_{F,s} = \arg \max_{K_E} \tilde{H}_s(K_E; \gamma).$$

(9)

In the following proposition we compare optimal investment levels and spinoff formation under internal provision with the case of external provision of complementary assets, and with the efficient benchmark.

**Proposition 8**

(i) If spinoff formation is efficient, then it always occurs under internal provision of the required complementary assets.

(ii) The optimal level of R&D effort with internal provision is strictly decreasing in $\gamma$ and always larger than the optimal level of R&D without internal provision, but below the efficient level $K_{F,eff}$.

As it is to be expected, the possibility of internal provision improves the ‘efficiency properties’ of the outcome. More precisely, we reach the efficient outcome in terms of spinoff formation, and investment levels increase compared to the scenario in which the incumbent lacks the assets to enable a spinoff. However, they fall short of the efficient amount.

The efficiency of spinoff formation is related to the fact that the firm and the employee can freely decide upon market structure without loosing any of the industry profits to a third party. Hence, they always choose the efficient structure, which in our case is one with spinoff formation. When the potential start-up has to rely on an external provider of complementary assets, choosing to form a start-up implies a loss of parts of the overall surplus to a third party, and therefore the firm and the employee may agree not to do so even if this is not efficient.

There are two reasons for an inefficient level of investment to arise. First, even with internal provision, the firm appropriates only a fraction $\xi$ of the generated overall surplus: a version of the standard hold-up problem that remains unsolved when internal provision is allowed for. Second, investment improves the outside options of the employee, causing a reduction on the incumbent’s incentives to invest ex ante, and hence reducing the net surplus. It is worth stressing that the latter effect becomes stronger whenever the availability of complementary assets increases, which explains why (even with internal provision) investment levels are decreasing in $\gamma$.

Finally, to understand why investment incentives improve compared to the case in which the incumbent can not enable a spinoff, it is useful to distinguish between the situation in which, without internal provision, a start-up is formed and that in which it is not. In the former, allowing for internal provision shifts part of the profits generated by the spinoff from the external provider to the incumbent; hence, the incentives to invest increase. In the latter, the results follow directly from the observation that the marginal return on investment are larger if a spinoff is formed.
6 Policy Implications and Concluding Remarks

The agenda of this paper is to study the effect of the availability of complementary assets on the process of start-up formation and on the R&D investments of potential parent firms. Both these issues have efficiency implications. As our focus is on the supply side, with a slight abuse of notation, we refer to deviations from maximal total industry profits as inefficiencies. As already noted two different types of inefficiencies may arise in our framework. If limited appropriability of returns to R&D investment by the incumbent leads to under-investment, we face ex-ante inefficiencies occurring before the decision about start-up formation is made. If no start-up is created although total industry profits could be increased by founding one, ex-post inefficiencies occur. Of course both types of inefficiencies might also occur simultaneously.

The policy debate focuses to a large extent on ex-post inefficiencies (too few start-ups), hence calling for policies directed at increasing the availability of complementary assets. Interpreting our results with respect to these efficiency considerations more refined implications arise. Standard hold-up arguments establish that the formation of start-ups always lead to ex-ante inefficiencies. However, we have shown that the amount of under-investment depends crucially on parameters that can be influenced by policy interventions. Furthermore, ex-post inefficiencies disappear for certain parameter settings. More precisely, if we consider the scenario in which internal provision is not an option, both ex-ante and ex-post inefficiencies occur for low values of $\gamma$. If the availability of complementary assets increases, the R&D investment of the incumbent decreases, and therefore ex-ante inefficiencies become more pronounced. However, if the increase in $\gamma$ is sufficiently strong to exceed the threshold $\bar{\gamma}$, ex-post inefficiencies disappear. A further increase of $\gamma$ has no influence on either R&D investment, or the start-up generation process. Put differently, when complementary assets are scarce, a gradual increase in their availability has strictly negative effects on efficiency, whereas when complementary assets are abundant adding more of them has no effects. For the intermediate cases, the way efficiency changes depends on the effects of R&D expenses. If R&D investment has a slow market expanding effect, then an increase of $\gamma$ across the $\bar{\gamma}$ threshold leads to the elimination of ex-post inefficiencies and to a reduction of the ex-ante under-investment. Accordingly, in this case a policy aimed at increasing the availability of complementary assets has indeed a clear-cut positive impact. In the case of a fast market expanding effect, a trade-off has to be faced in the sense that increasing $\gamma$ across $\bar{\gamma}$ eliminates ex-post inefficiencies but worsens the problem of the ex-ante under-investment. Therefore, general statements about the size of the overall impact of an increase in $\gamma$ on total profits can not be made without being more specific about the demand structure.

Recalling the economic interpretations of the parameter $\gamma$ discussed in Section 2, our results suggest that in markets in which incumbent firms have sufficient market

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8It should be noted that reverse ex-post inefficiencies, in the sense that a start-up is formed although it is detrimental for industry profits, never occur in our setup since we only consider scenarios in which start-up formation increases total profits.

9Measures aimed at improving the development of private equity markets, and programs directed at creating incubators or innovation parks, are primary examples of policies geared toward increasing the availability of assets required for successful start-up formation.
power to restrict access of potential entrants to key assets (e.g. software industry, energy trading), a relatively small number of start-ups should be observed, while the investment of the dominant firms in innovative activities should be relatively high. This argument supports the Schumpeterian view that market concentration is positively correlated to the intensity of innovative activities.

In the opposite case, in which complementary assets become ‘freely’ available for potential start-up founders (i.e. $\gamma$ approaches one), ex-post inefficiencies disappear, but there is significant ex-ante under-investment. These conclusions hold true no matter whether strong competition among providers of complementary assets shifts all the bargaining power to the start-up founder, or the availability of complementary assets is due to government programs facilitating start-up formation. Accordingly, our findings give a direct indication of the impact of policy programs that make the needed complementary assets available to potential start-up founders. As we have shown, the introduction of such programs may have a positive impact by reducing both ex-ante and ex-post inefficiencies, but under certain circumstances it may diminish the R&D investments of incumbents.

Until now our discussion has focused on the role of the availability of complementary assets. A second parameter that can be influenced by policy measures is the bargaining power $\xi$ of the firm when negotiating with its R&D employee. Under the assumption that this parameter is mainly determined by the firm’s options to replace the key R&D employee, a high value of $\xi$ can be seen as the expression of a strong supply of qualified R&D employees in the labor market. In this perspective, our results indicate that an increase in the supply of R&D employees moves the threshold $\gamma$ to the right, and therefore makes it more likely that no start-up is formed and ex-post inefficiencies occur. Whether the increase in ex-post inefficiencies is paired with positive or negative effects with respect to ex-ante under-investment depends on whether R&D investment has a fast or slow market expanding effect.

On the one hand, the observation that policies leading to an increase in the availability of complementary assets and in the supply of potential R&D employees may have detrimental effects on innovative activities and industry profits indicates that these policies should be applied with caution. On the other hand, the observations made within the framework of this model should be treated with appropriate reservations. As already stated in the Introduction, policy measures such as those discussed above have a number of effects that lie outside the scope of this paper, such as the facilitation of industry agglomeration, the improved productivity of future R&D investments, or the future improvement of knowledge flows in the industry. Many of these effects positively influence R&D activities and industry profits, thereby generating trade-offs with the effects discussed here. In this paper we do not provide a complete analysis of these trade-offs, but we point out that policies – typically regarded as innovation-friendly – can be associated with negative strategic effects.

A main drawback of our static model structure is that the dynamic effects of start-up formation on the evolution of competition and industry structure, as well as future knowledge flows, can not be captured. Developing and analyzing models combining

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10 In our model this situation corresponds to a small value of $\gamma$.
11 Note that as far as ex-post inefficiencies are concerned the possibility of internal financing corresponds to the limit case in which $\gamma$ approaches one.
dynamic features with the strategic effects considered here is a challenging area for future research. Furthermore, the predictions of our analysis concerning the effects of the availability of complementary assets on incumbents' R&D investments could be the theoretical starting point for empirical and experimental work.

References


Appendix A

In this Appendix we use a quality-augmented duopoly model with linear demand (see Symeonidis (2003)) to qualitatively exemplify the effects characterized in our general analysis and gain a better understanding of the differences between slow and fast market expanding effects.

We consider a market in which, without start-ups, the incumbent acts as a monopolist. Consumers have quality augmented Dixit-Stiglitz-type utility functions. The corresponding inverse market demand is given by

\[ p_{m}^{F} = 1 - 2 \frac{q_{m}^{F}}{u_{F}^{2}}, \]

where \( u_{F}^{m} \) denotes the quality of the good offered. Marginal costs are constant and denoted by \( 0 < c_{F} < 1 \). The optimal behavior of the monopolist yields the profit

\[ \bar{\pi}_{F}^{m} = \frac{(1 - c_{F})^{2} u^{2}}{8}. \]
We assume that product quality is related to the knowledge $K_E$ by

$$u_F^m = K_E^\mu, \mu > 0,$$

and the investment needed to generate $K_E$ is given by

$$I(K_E) = \iota K_E^\nu, \nu > 1, \iota > 0.$$

Accordingly, the profit function reads

$$\pi^m_F(K_E) = \tilde{\pi}_F^m = \frac{(1 - c_F)^2 K_E^{2\mu}}{8}. \quad (10)$$

In case a start-up is formed, the quality of the product the incumbent’s can offer on the market is only

$$u_F^s = (\delta K_E)^\mu, \quad (11)$$

whereas the quality of the start-up’s product is

$$u_E^s = K_E^\mu. \quad (12)$$

Furthermore, the two products are horizontally differentiated. Denoting by $(p_F^s, p_E^s)$ and $(q_F^s, q_E^s)$ the prices and quantities of the incumbent and of the start-up in case a start-up is generated, the inverse demand system reads

$$p_F^s = 1 - 2 \frac{q_F^s}{u_F^s} - \frac{\sigma q_E^s}{u_F^s u_E^s},$$

$$p_E^s = 1 - 2 \frac{q_E^s}{u_E^s} - \frac{\sigma q_F^s}{u_F^s u_E^s}.$$ 

The parameter $\sigma \in [0, 2]$ determines the degree of horizontal differentiation, where lower $\sigma$ values correspond to a higher degree of differentiation. The constant marginal production costs of the start-up are denoted by $c_E$ and we assume that, due to learning curve effects, the start-up’s marginal production costs are larger or equal than those of the incumbent, i.e. $c_E \geq c_F$. Assuming that the two firms engage in quantity competition, standard calculations yield the following profits in the Cournot equilibrium:

$$\tilde{\pi}_F^s = 2 \left( \frac{4(1 - c_F)u_F^s - \sigma(1 - c_E)u_E^s}{(4 - \sigma)(4 + \sigma)} \right)^2,$$

$$\tilde{\pi}_E^s = 2 \left( \frac{4(1 - c_E)u_E^s - \sigma(1 - c_F)u_F^s}{(4 - \sigma)(4 + \sigma)} \right)^2.$$ 

Inserting (11) and (12), we obtain

$$\pi_F^s = 2K_E^{2\mu} \frac{(4(1 - c_F)\delta^\mu - \sigma(1 - c_E))^2}{(4 - \sigma)^2(4 + \sigma)^2}, \quad (13)$$

$$\pi_E^s = 2K_E^{2\mu} \frac{(4(1 - c_E) - \sigma(1 - c_F)\delta^\mu)^2}{(4 - \sigma)^2(4 + \sigma)^2}. \quad (14)$$
From (10) and (13), we get
\[
\Delta \pi_F = 2K_E^{2\mu} \left( \frac{(1 - c_F)^2}{16} - \frac{(4(1 - c_F)\delta - \sigma(1 - c_E))^2}{(4 - \sigma)^2(4 + \sigma)^2} \right) \tag{15}
\]
For the sake of simplicity, we assume also that the cost of providing the complementary assets required to found a start-up is a polynomial in \(K_E\),
\[
M_s(K_E) = aK_E^\lambda,
\]
which gives
\[
\Delta \pi_E = 2K_E^{2\mu} \frac{4(1 - c_E) - \sigma(1 - c_F)^2}{(4 - \sigma)^2(4 + \sigma)^2} - aK_E^\lambda. \tag{16}
\]
It is easy to check that these functional forms satisfy all assumptions made in our general analysis.

The elasticity of \(\Delta \pi_F\) is constant and given by
\[
e_F(K_E) = 2\mu, \tag{17}
\]
whereas we obtain for the elasticity of \(\Delta \pi_E\)
\[
\epsilon_E(K_E) = \frac{4\mu K_E^{2\mu} (4(1 - c_E) - \sigma\delta^2(1 - c_F))^2 - (4 - \sigma)^2(4 + \sigma)^2 a K_E^\lambda}{2K_E^{2\mu} (4(1 - c_E) - \sigma\delta^2(1 - c_F))^2 - (4 - \sigma)^2(4 + \sigma)^2 a K_E^\lambda} = \frac{2\mu - a K_E^\lambda}{2K_E^{2\mu} (4(1 - c_E) - \sigma\delta^2(1 - c_F))^2 - (4 - \sigma)^2(4 + \sigma)^2 a K_E^\lambda}.
\]
It is then easy to conclude that \(\epsilon_F(K_E) \leq \epsilon_E(K_E)\) if and only if
\[
\mu \geq \frac{\lambda}{2}. \tag{18}
\]
It should be noted that the Condition (18) for a slow market expanding effect is independent from the values of all demand and cost parameters, as it depends only on the elasticity of the product quality and on the cost of providing the complementary assets with respect to the stock of knowledge \(K_E\). If the elasticity of product quality with respect to knowledge is high relative to the elasticity of the costs of complementary assets, profits on the market (monopoly and duopoly profits) are growing with \(K_E\) at a higher rate than the costs of the required complementary assets. Accordingly, the marginal positive effects of knowledge investment on net industry profits are smaller for low than for high investment levels, and we have a slow market expanding effect of investments. For small values of \(\mu\) these effects are reversed and we have a fast market expanding effect.

Figures 2 - 5 are based on the linear example described above with parameter values
\[
a = 0.005, \ \lambda = 1.5, \ \nu = 0.05, \ \sigma = 3, \ c_F = 0.2, \ c_E = 0.25, \ \delta = 0.9, \ \xi = 0.5.
\]
The condition for a slow market expanding effect is \(\mu \geq 0.75\): we set \(\mu = 1\) to illustrate cases with a slow market-expanding effect, and \(\mu = 0.5\) for the cases with a fast market-expanding effect.
Appendix B

Proof of Lemma 1
(i) Follows directly from (6).
(ii) The first claim follows directly from \( \frac{\partial K_{E,m}}{\partial \gamma} = -\frac{\partial^2 H_m}{\partial \gamma^2} \), the concavity of \( H_m \) and (6). The second is obvious as \( \bar{K}_{E,s} \) does not depend on \( \gamma \).
(iii) We show that \( \bar{K}_{E,m} > \bar{K}_{E,s} \) for \( \gamma = 0 \) and \( \bar{K}_{E,m} < \bar{K}_{E,s} \) for \( \gamma = 1 \). If \( \gamma = 0 \) then \( \frac{d(H_m - H_s)}{d\bar{K}_E} = \xi \xi' \{ \bar{K}_E; 0 \} \) by Assumption 2. Hence, \( \frac{d(H_m(\bar{K}_{E,s}; 0))}{d\bar{K}_E} > 0 \), which implies \( \bar{K}_{E,m} > \bar{K}_{E,s} \). If \( \gamma = 1 \) then \( \frac{d(H_m - H_s)}{d\bar{K}_E} = -\xi \xi' \{ \bar{K}_{E,s}; 1 \} < 0 \) again by Assumption 2. Hence, \( \frac{d(H_m(\bar{K}_{E,s}; 1))}{d\bar{K}_E} < 0 \), which implies \( \bar{K}_{E,m} < \bar{K}_{E,s} \). The third claim follows from the monotonicity of \( \bar{K}_{E,m} - \bar{K}_{E,s} \) with respect to \( \gamma \).
(iv) Consider an arbitrary \( \xi \in (0, 1) \). For \( \gamma = \hat{\gamma} \) we have \( \bar{K}_{E,m} = \bar{K}_{E,s} \). Therefore, \( H_m'(\bar{K}_{E,s}; \hat{\gamma}) = H_s'(\bar{K}_{E,s}; \hat{\gamma}) = 0 \), which implies

\[
H_m'(\bar{K}_{E,s}; \hat{\gamma}) - H_s'(\bar{K}_{E,s}; \hat{\gamma}) = -\xi \xi' \{ \bar{K}_{E,s}; \hat{\gamma} \} = 0.
\]

Accordingly, it must hold that \( G'(\bar{K}_{E,s}; \hat{\gamma}) = 0 \). Since \( \bar{K}_{E,s} \) does not depend on \( \xi \) this implies that \( H_m'(\bar{K}_{E,s}; \gamma) = 0 \) for all \( \xi \in (0, 1) \) and therefore \( \hat{\gamma} \) does not depend on \( \xi \).

Proof of Proposition 2
(i) \( K^*_E(\gamma) = \bar{K}_{E,m} \) in SPNE if and only if \( H_m(\bar{K}_{E,m}; \gamma) - H_s(\bar{K}_{E,s}) \geq 0 \). Notice that, for \( \gamma = 0 \), we have

\[
H_m(\bar{K}_{E,m}; 0) - H_s(\bar{K}_{E,s}) \geq H_m(\bar{K}_{E,s}; 0) - H_s(\bar{K}_{E,s}) = \xi \Delta \pi_F(\bar{K}_{E,s}) > 0,
\]

where the last inequality follows from Assumption 2. Furthermore, using the envelope theorem we get

\[
\frac{d}{d\gamma} \left( H_m(\bar{K}_{E,m}; \gamma) - H_s(\bar{K}_{E,s}) \right) = \frac{\partial}{\partial \gamma} \left( H_m(\bar{K}_{E,m}; \gamma) - H_s(\bar{K}_{E,s}) \right) = \frac{\partial H_m(\bar{K}_{E,m})}{\partial \gamma} < 0.
\]

If \( H_m(\bar{K}_{E,m}; 1) - H_s(\bar{K}_{E,s}) > 0 \) we set \( \hat{\gamma} = 1 \) and \( K^*_E(\gamma) = \bar{K}_{E,m} \) for all \( \gamma \in (0, 1] \). Otherwise, there exists a unique \( \hat{\gamma} \in (0, 1] \) such that \( H_m(\bar{K}_{E,m}; \hat{\gamma}) - H_s(\bar{K}_{E,s}) = 0 \), and the first claim of the proposition follows.
(ii) From the definition of \( \bar{K}_{E,m} \) and \( K_{E}^{eff} \), and from

\[
\frac{\partial}{\partial K_E} (H_m(K_E; \gamma) - (\pi_F^s(K_E) + \pi_E^s)(K_E) - I(K_E))
\]

\[
= \xi \Delta \pi_F(K_E) - (\xi \gamma + 1) \Delta \pi_E(K_E) < 0
\]

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it follows that $\bar{K}_{E,m} < K_E^{eff}$. Analogously, we obtain $\bar{K}_{E,s} < K_E^{eff}$. These two inequalities combined lead to $K_E^*(\gamma) < K_E^{eff}$.

\[2\]

Proof of Proposition 3
Consider first the case in which knowledge has a slow market expanding effect. Taking into account Lemma 1 and Proposition 2, all we have to show is that $\bar{\gamma} > \hat{\gamma}$. Note first that under the assumption of market expanding knowledge we have

$$\frac{\Delta \pi'_{F}(K_E)}{\Delta \pi_{E}(K_E)} < \frac{\Delta \pi_{F}(K_E)}{\Delta \pi_{E}(K_E)}$$

(19)

for all $K_E > 0$. Furthermore, it has been shown in the proof of Lemma 1 (iv) that $G'(\bar{K}_{E,s}; \hat{\gamma}) = G'(\hat{K}_{E,m}(\hat{\gamma}); \hat{\gamma}) = 0$, which implies

$$\hat{\gamma} = \frac{\Delta \pi'_{F}(\bar{K}_{E,s})}{\Delta \pi_{E}(\bar{K}_{E,s})}.$$

Hence,

$$H_m(\bar{K}_{E,m}(\hat{\gamma}); \hat{\gamma}) - H_s(\bar{K}_{E,s})$$

$$= H_m(\bar{K}_{E,s}; \hat{\gamma}) - H_s(\bar{K}_{E,s})$$

$$= \xi \left( \Delta \pi_{F}(\bar{K}_{E,s}) - \hat{\gamma} \Delta \pi_{E}(\bar{K}_{E,s}) \right)$$

$$= \xi \left( \Delta \pi_{F}(\bar{K}_{E,s}) - \frac{\Delta \pi'_{F}(\bar{K}_{E,s})}{\Delta \pi_{E}(\bar{K}_{E,s})} \Delta \pi_{E}(\bar{K}_{E,s}) \right)$$

$$> 0,$$

where the last inequality follows from (19). Accordingly, for $\gamma = \hat{\gamma}$ no start-up is formed in the SPNE, and we must have $\bar{\gamma} > \hat{\gamma}$.

The proof that $\bar{\gamma} < \hat{\gamma}$ when knowledge has a fast market expanding effect proceeds along the same lines as above, where it has to be taken into account that if knowledge has a fast market expanding effect then

$$\Delta \pi_{F}(\bar{K}_{E,s}) - \frac{\Delta \pi'_{F}(\bar{K}_{E,s})}{\Delta \pi_{E}(\bar{K}_{E,s})} \Delta \pi_{E}(\bar{K}_{E,s}) < 0.$$

The claim concerning $\bar{K}_{E,m}$ follows directly. \[2\]

Proof of Proposition 5
We first state and prove the following Lemma.

Lemma 9

(i) $P_{F}^{*}(\gamma)$ is single valued and continuous on $(0,1]$;

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(ii) $P_E^*(\gamma)$ is single valued and continuous for $\gamma \neq \tilde{\gamma}$.

**Proof**

(i). For $\gamma \neq \tilde{\gamma}$ there is a unique SPNE, and therefore $P_E^*(\gamma)$ is single-valued. For $\gamma = \tilde{\gamma}$, there are coexisting equilibria with and without start-up but, due to the definition of $\tilde{\gamma}$, the profit of the firm is the same in both equilibria. Hence, $P_E^*(\gamma)$ is single-valued and continuous.

(ii). The fact that $P_E^*(\gamma)$ is single-valued follows immediately from the uniqueness of SPNE for $\gamma \neq \tilde{\gamma}$. Continuity follows directly from the continuity of $P_E^{22}, P_E^{32}, P_E^{21}$ and $P_E^{31}$.

We can now turn to the proof of the proposition.

(i). Note that

$$P_E^*(\gamma) = \begin{cases} H_m(\bar{K}_{E,m}(\gamma); \gamma) & \gamma < \tilde{\gamma} \\ H_s(\bar{K}_{E,s}) & \text{otherwise.} \end{cases}$$

Using the envelope theorem we obtain for $\gamma < \tilde{\gamma}$ that

$$\frac{\partial P_E^*(\gamma)}{\partial \gamma} = \frac{\partial H_m(K_{E,m}(\gamma); \gamma)}{\partial \gamma} = -\xi \Delta \pi_E(K_{E,m}(\gamma)) < 0.$$ 

For $\gamma \geq \tilde{\gamma}$ the claim follows from the independence of $H_s$ from $\gamma$.

(ii). The claim follows directly from Equation 2, as for $\gamma > \tilde{\gamma}$ a start-up is formed in equilibrium, the firm’s investment level is constant in $\gamma$ and the corresponding employee’s payoff is given by $P_E^{22} = P_E^{32}$.

(iii). If $\gamma < \tilde{\gamma}$, the employee’s payoff is given by

$$P_E^*(\gamma) = \xi P_E^{21} + (1 - \xi) P_E^{31} = \gamma \Delta \pi_E^s(\bar{K}_{E,m}) - (1 - \xi) G(\bar{K}_{E,m}; \gamma).$$

Taking this into account we obtain

$$\lim_{\gamma \to \tilde{\gamma}^-} P_E^*(\gamma) = \tilde{\gamma} \Delta \pi_E^s(\bar{K}_{E,m}(\tilde{\gamma})) - (1 - \xi) G(\bar{K}_{E,m}(\tilde{\gamma}); \tilde{\gamma})$$

$$\lim_{\gamma \to \tilde{\gamma}^+} P_E^*(\gamma) = \tilde{\gamma} \Delta \pi_E^s(\bar{K}_{E,s}).$$

From Proposition 3, we know that if knowledge has a fast market expanding effect then $\bar{K}_{E,m}(\tilde{\gamma}) > \bar{K}_{E,s}$. Furthermore,

$$G(\bar{K}_{E,m}(\tilde{\gamma}); \tilde{\gamma})$$

$$= -\frac{1}{\xi}(H_m(\bar{K}_{E,m}(\tilde{\gamma}); \tilde{\gamma}) - H_s(\bar{K}_{E,m}(\tilde{\gamma}))$$

$$\leq -\frac{1}{\xi}(H_m(\bar{K}_{E,m}(\tilde{\gamma}); \tilde{\gamma}) - H_s(\bar{K}_{E,s})$$

$$= 0,$$
where the inequality follows from the definition of $\bar{K}_{E,s}$ and the last equality from the definition of $\bar{\gamma}$. Taking $G(\bar{K}_{E,m}(\bar{\gamma}); \bar{\gamma}) \leq 0$ into account, the strict monotonicity of $\Delta\pi_E$ with respect to $K_E$ establishes that

$$\lim_{\gamma \to \bar{\gamma}^-} P_E^*(\gamma) > \lim_{\gamma \to \bar{\gamma}^+} P_E^*(\gamma).$$

□

Proof of Lemma 6
By definition of $\bar{\gamma}$, we have $H_m(\bar{K}_{E,m}(\bar{\gamma}); \bar{\gamma}) - H_s(\bar{K}_{E,m}(\bar{\gamma})) = 0$. Implicit differentiation of $H_m(\bar{K}_{E,m}; \bar{\gamma}) - H_s(\bar{K}_{E,m}) = 0$ with respect to $\xi$ yields

$$\frac{\partial \bar{\gamma}}{\partial \xi} = -\frac{G(\bar{K}_{E,m}(\bar{\gamma}); \bar{\gamma})}{\xi \Delta\pi_E(\bar{K}_{E,m}(\bar{\gamma}))} = \frac{H_m(\bar{K}_{E,m}(\bar{\gamma}); \bar{\gamma}) - H_s(\bar{K}_{E,m}(\bar{\gamma}))}{\xi^2 \Delta\pi_E(\bar{K}_{E,m}(\bar{\gamma}))}.$$

The denominator of this expression is positive. Concerning the numerator, we observe that for all $\bar{\gamma} \neq \hat{\gamma}$ we have $\bar{K}_{E,m}(\bar{\gamma}) \neq \bar{K}_{E,s}$, and therefore

$$H_m(\bar{K}_{E,m}(\bar{\gamma}); \bar{\gamma}) - H_s(\bar{K}_{E,m}(\bar{\gamma})) > H_m(\bar{K}_{E,m}(\hat{\gamma}); \hat{\gamma}) - H_s(\bar{K}_{E,s}) = 0.$$ 

This implies, $\frac{\partial \bar{\gamma}}{\partial \xi} > 0$ for $\bar{\gamma} \neq \hat{\gamma}$ and $\frac{\partial \bar{\gamma}}{\partial \xi} = 0$ for $\bar{\gamma} = \hat{\gamma}$. □

Proof of Proposition 7
Denote by $\tilde{\gamma}$ the limit of $\gamma$ for $\xi \to 0$. Since $\tilde{\gamma}$ is monotonic with respect to $\xi$ (Lemma 6) and has to be in $[0,1]$ for all $\xi > 0$ the limit exists and $\tilde{\gamma} \in [0,1]$. Furthermore, for any $\gamma < \tilde{\gamma}$ we must have $\gamma < \tilde{\gamma}$ for all $\xi \in (0,1)$, which implies by Proposition 2 that no start-up is formed. On the other hand, if $\gamma > \tilde{\gamma}$ then by monotonicity of $\tilde{\gamma}$ with respect to $\xi$, there must either exist a $\xi \in (0,1]$ such that $\tilde{\gamma}(\xi) = \gamma$ for $\xi = \xi \gamma$ or $\tilde{\gamma}(\xi) < \gamma$ for all $\xi \in [0,1]$, where we set $\xi = 1$ in the latter case. We obtain that $\tilde{\gamma}(\xi) \leq \gamma$ if and only if $\gamma \leq \xi$ and the claim of the Proposition follows from Proposition 2. □

Proof of Proposition 8
(i). Follows directly from the discussion in the text.
(ii). To check that $\bar{K}_{E,s}$ is decreasing in $\gamma$ one just has to verify that

$$\frac{\partial^2 \bar{H}_s(K_E, \gamma)}{\partial K_E \partial \gamma} = -\xi \Delta\pi_E(K_E)$$
is negative, which follows from Assumption 2.

In order to show that \( \tilde{K}_{F,s} > K^*_F \) for all \( \gamma \in (0, 1) \), note first that

\[
\tilde{H}_s(K_E; \gamma) - H_s(K_E; \gamma) = \xi(1 - \gamma)\Delta\pi_E(K_E).
\]

By Assumption 2, we have

\[
\frac{\partial(\tilde{H}_s(K_E; \gamma) - H_s(K_E; \gamma))}{\partial K_E} = \xi(1 - \gamma)\Delta\pi_E'(K_E) > 0.
\]

This implies \( \tilde{K}_{F,s} > \tilde{K}_{E,s} \). Furthermore,

\[
\tilde{H}_s(K_E; \gamma) - H_m(K_E; \gamma) = \xi(\Delta\pi_E(K_E) - \Delta\pi_F(K_E)),
\]

which again by Assumption 2 implies \( \tilde{K}_{F,s} > \tilde{K}_{E,m} \). Hence, \( \tilde{K}_{F,s} > K^*_F \) for all \( \gamma \in (0, 1) \).

Finally, \( \tilde{K}_{F,s} < K^{eff}_E \) follows by taking into account the definition of \( \tilde{K}_{F,s}, K^{eff}_E \) and noting that direct calculation shows that

\[
\frac{\partial(\tilde{H}_s(K_E; \gamma) - (\pi_{F,s}(K_E) + \pi_{E,s}(K_E) - I(K_E)))}{\partial K_E} = (\xi(1 - \gamma) - 1)\Delta\pi_E'(K_E) < 0.
\]

\( \square \)
Elenco Quaderni già pubblicati


66. L. Colombo, M. Grillo, Collusion when the Number of Firms is Large, marzo 2006.
69. A. Baglioni, Entry into a network industry: consumers’ expectations and firms’ pricing policies, novembre 2006.
75. A. Baglioni, Corporate Governance as a Commitment and Signalling Device, novembre 2007.

