HETEROGENEOUS AGENTS, INDEXATION AND THE NON NEUTRALITY OF MONEY

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Summary

Some relevant macroeconomic results, such as the money-neutrality proposition, usually rest on the representative agent assumption. Those results, however, are not robust since an even small behavioural heterogeneity (e.g. a few agents are near-rational) leads to non-neutrality. The paper shows that perfect-indexation policies, aimed at neutralising nominal shocks, may be very information demanding under heterogeneity and that rule-of-thumb behaviour on the part of rational agents and the government may deliver better results than fully rational behaviour.

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I. INTRODUCTION

Several attempts have been made in recent economic literature to abandon the representative individual approach and to explore the macroeconomic implications of behavioural heterogeneity\(^1\). In two pioneering works Akerlof and Yellen (1985a, 1985b) showed that even such a small deviation from the representative individual framework as the presence of some near-rational firms leads to real effects of monetary shocks in a model where full rationality on the part of all firms would lead to money-neutrality.

In a sequence of papers Haltiwanger and Waldman (1985, 1989, 1991) examined the relations between near-rationality and the nature of strategic interactions among economic agents. They were able to show that strategic complementarity causes near-rational agents to be disproportionally important. In the same vein Haltiwanger and Waldman (1989) proved that under strategic complementarity agents who have non-rational expectations tend to be disproportionally important in determining a slow macroeconomic adjustment path after a one-time shock and that the higher the share of agents with non-rational expectations the higher the degree of output persistence following the shock\(^2\).

The intuition behind these findings is simple. Due to strategic complementarity the presence of near-rational agents incentives fully rational agents to bias their behaviour towards that of the near-rational ones; and such an incentive is positively related to the degree of strategic complementarity and to the share of near-rational agents in the population.

As the payoffs of optimisers are greater than those of non-optimisers, one may legitimately suspect that in a multiperiod setting the population composition will evolve in favour of the group with the higher payoffs and the fraction of near-rational agents will be driven to extinction in the long run.

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1 A forceful critique of models built on the representative individual assumption can be found in Kirman (1992).
2 A similar result has been proved by Andersen and Hviid (1995), under the assumption that there are some firms that are informed and some that are uninformed about the actual realisation of a monetary shock, whilst Bonf\'in and Diebold (1997) extended Haltiwanger and Waldman (1991) to a fully dynamic economy which is hit by both real and monetary shocks.
In a recent paper Sethi and Franke (1995) show that the existence of an optimisation cost³ makes it possible for non-optimisers to survive and coexist with the optimisers under very general conditions. In fact, Sethi and Franke are able to prove that it is the survival of sophisticated optimising agents that is conditional on the existence of some exogenous variability in the environment⁴.

As the persistent heterogeneity of economic agents can be derived from evolutionary dynamics, for the purpose of the present paper we feel allowed to simply assume the existence of a given fraction of near-rational agents.

In fact, the aim of the paper is not to build a realistic dynamic macroeconomic-policy model, nor to discuss the foundations of the policy maker's objective function. Rather, we are interested in the logical possibility of pursuing a given government intervention, whatever the rationales behind authorities' choices, in an environment where behavioural heterogeneity is combined with strategic complementarity. In what follows, we assume, for the sake of simplicity, that the policy maker's objective function is to neutralise the effects of monetary shocks on the aggregate level of production and employment.

Furthermore, we introduce the hypothesis that behavioural heterogeneity is represented - as in Haltiwanger and Waldman (1991) - by the presence of two types of agents: ‘responders’ and ‘non-responders’: fully rational responders optimally adjust to nominal shocks, whilst near rational non-responders do not adjust at all. Responders know with certainty the proportion of non-responders in the population, whilst non-responders simply ignore the existence of responders (their behaviour is independent of the actions chosen by responders).

It turns out that, with heterogeneous agents, more information is needed to construct neutralising reaction rules to other agents’ behaviour. The presence of near-rational agents not only disproportionately affects the macroeconomic outcome - as emphasised by Haltiwanger and Waldman - but also influences the ability of the policy maker to pursue his stabilisation objective, as

³ As Sethi and Franke (1995, p. 584) write optimisation cost "is the one cost that cannot in principle be fully accommodated in an optimisation model", because of the infinite regress in which the modeler would inevitably be caught.

⁴ By means of numerical simulations, Sethi and Franke (1995) were also able to show that Haltiwanger and Waldman's positive relation between the persistence of real effects of nominal shocks and the share of near-rational agents can be extended to a fully dynamic context: ‘the finding that a higher long-run share of naive agents is associated with greater serial correlation is a confirmation and generalisation of earlier work’ (p.595).
the environment in which he and fully rational agents act is significantly altered by heterogeneity. In order to counterbalance the shocks, the policy maker has to know the degree of non responsiveness in the model economy and the impact of non responsiveness on the macroeconomic outcome, while rational agents, in turn, must form accurate expectations about actual behaviour of non responders and about the perception of the degree of non responsiveness on the part of the policy maker.

The presence of near rational agents significantly modifies the information requirements for all other agents. In these circumstances, if rational agents and policy makers act as if they ignore the existence of near-rational agents, better results are obtained in terms of macroeconomic objectives, since macroeconomic fluctuations may be hampered with respect to situations in which rational agents try, but fail, to make use of their knowledge. Moreover, it will also be shown that rational agents may find it profitable to ignore the presence of non responder agents, and no individual incentive may induce rational firms to form accurate predictions and to adopt more sophisticated behavioural rules.

The paper is organised as follows. In section 2 a simple macro-model is set up, featuring monopolistic competition, with aggregate demand only acting through real money balances. Following Dixon (1990), wages are set by (fully rational) monopoly unions as a mark-up over the unemployment benefit, hence an indexation problem of such a benefit arises if shocks are to be neutralised. In order to keep the analytical complexity to a minimum, it is assumed that fully rational agents have perfect foresight.

Were all firms fully rational responders the indexation problem is easily solved as the policy maker knows that all firms will optimally adjust their prices to the shock. In this case it can be shown (section 3) that full indexation to the price level reduces to indexing the unemployment benefit to the monetary shock (which is perfectly anticipated by the policy maker). No more information than the

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5 The model employed differs from the one in Akerlof and Yellen (1985b) for not featuring efficiency wages. The monopoly union assumption has been chosen only as a simple way for introducing an indexation problem.

6 See Dixon, Rankin (1994) for a discussion of the class of models to which the one presented here belongs.

7 There are understandable reasons for implementing an indexation policy in order to neutralise nominal shocks. Such a policy, extensively analysed in Dixon (1990), offers the advantage that can be adopted timely and at a low cost.
initial price level and the magnitude of the shock is needed, provided the policy maker knows that all firms are fully rational. A simple rule is also neutralising and standard monetarist predictions apply.

The main results of the paper are presented in sections 4 and 5. Proposition 1 shows that the presence of a small number of near-rational non-responders makes any full indexation rule unable to neutralise a one-time monetary shock. It will also be shown that, under full indexation, the proportion of near-rational agents and the degree of real flexibility positively affect the distance between equilibrium and target outcomes after a shock. The existence of non-responders makes the standard full-indexation rule ineffective. Moreover, the full-indexation rule under heterogeneity requires that the policy maker knows every structural parameter of the economy, whilst fully rational agents must be supposed to be able to know in advance the indexation rule (Proposition 2). Proposition 3 shows that neutralisation of monetary shocks requires an over-indexation of the unemployment benefit, according to a rule (which will be called “perfect indexation”), which is at least as information demanding as the full-indexation rule mentioned above. Over-indexation implies that the real wage is no longer constant. It is actually the pro-cyclical variation of the real wage that hampers the real impact of the shock up to its complete neutralisation. Proposition 4 shows the rather paradoxical result that a simple rule such as indexing the unemployment benefit to the monetary shock, despite being far less demanding in terms of information acquisition and processing abilities, is more effective than a full-indexation rule in neutralising monetary shocks, though less effective than “perfect indexation”. Finally, Proposition 5 shows that, provided the policy maker is not able to determine a perfect indexation rule, the model-economy exhibits less responsiveness to nominal disturbances when responder firms ignore the presence of near-rational firms than when responder firms try to optimally react to the presence of near-rational firms.

Section 6 presents some concluding remarks and suggests that more difficulties would arise under heterogeneity were the deterministic model employed be replaced by a stochastic one.

II. THE MODEL
Assume that each price-making firm (their number is normalised to 1) operating in a goods market with monopolistic competition, faces the by now standard demand function:

\[ y_i = y \left( \frac{P_i}{P} \right)^{-\theta} = \frac{M}{P} \left( \frac{P_i}{P} \right)^{-\theta} \]  

(1)

where \( y \) is overall output, the number of firms (= number of goods produced) is normalised to 1, \( P_i \) is the price charged by the \( i \)-th firm, \( P \) is the general price level, \( M \) is the money supply and \( \theta > 1 \).

Firms have a Cobb-Douglas production function such as:

\[ y_i = n_i^\alpha \quad \alpha \leq \beta \]  

(2)

The wage is fixed by an industry-wide monopolistic union representing \( \bar{n}_i \) identical workers and with a utilitarian objective function (Oswald 1985) such as:

\[ V_i(w_i, n_i) = \begin{cases} \frac{n_i (w_i - b)}{n_i (w_i - b)} & \text{for } n_i \leq \bar{n}_i \\ \frac{n_i (w_i - b)}{n_i (w_i - b)} & \text{for } n_i > \bar{n}_i \end{cases} \]  

(3)

where \( V_i \) is the trade union’s utility function, \( b \) is the real reservation wage. Sectoral unions assume the general price level as given and take the nominal unemployment benefit \( (B) \) to be their minimum reference wage. For the sake of simplicity, we assume that the real unemployment benefit \( \left( \frac{B}{P} \right) \) is the only alternative to the wage deriving from employment in whatever firm; i.e. that the chances of finding a new job after dismissal are nil. Each union maximises its utility function under the condition that the firm is on his labour demand curve.

In equilibrium the real wage, if the constraint on employment is not binding \( (n_i \leq \bar{n}_i) \), is set as a mark-up over the unemployment benefit:

\[ \frac{W_i}{P} = \gamma \frac{B}{P} \]  

(4)

where: \( \gamma = \frac{1}{\beta} \) and \( \beta = \frac{\alpha(\theta - 1)}{\theta} \).

When this economy is hit by a nominal shock only an indexation of \( B \) may prevent real effects on output and employment to take place. In principle, as agents’ market power leads to an

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8 See, for example, Blanchard, Fisher (1989), chapters 8 and 9.
9 See Dixon (1990) for the derivation of equation (4).
inefficiently low equilibrium level of output and employment, the neutralisation of monetary shocks may be a questionable objective (Benassi, Chirco, Colombo, 1997, 62-66). In fact, while negative demand shocks determine welfare losses positive shocks enhance welfare. The policy maker should thereby adopt contingent rules. However, non-contingent rules may be justified on the ground that (i) “the utility costs of the increased labour supply in a boom would be largely offset the benefits of the increased production, while the gains from the increased leisure in a recession would be small relative to the costs of the lost production” (Romer, 1993, 14); (ii) income stability may have positive influence on investment decisions of firms (Meltzer, 1988); (iii) contingent indexation rules such that nominal wages are flexible downwards but rigid upwards would have scanty chances to be accepted by trade unions and workers.\(^{11}\)

In what follows the preceding arguments for non-contingent rules are accepted and the assumption is made that price stability has a zero-weight in the government objective function, while output stability has a weight equal to one.

**III. THE IMPACT OF A SHOCK UNDER HOMOGENEOUS FULL RATIONALITY**

It is a standard result in this kind of models that if the unemployment benefit is fully indexed to the price level and all firms are fully rational, i.e. perfectly respond to shocks, money neutrality applies.\(^{12}\) To see this suppose that a monetary shock of magnitude \(\mu\) occurs such that after the shock \(M = M_0 (1 + \mu)\), where \(M_0\) is the money supply before the shock, hence \(d \log M = \log (1 + \mu)\).

The profit of the \(i\)-th firm (\(\pi_i\)) can be expressed as a function of the price \(P_i\) charged by the firm (the individual strategy), the general price level (the strategies of the other firms) and the money supply (the authorities control variable):

\[
\pi_i = \pi(P_i, P, M) \quad (5)
\]

\(^{10}\) One may suppose, for instance, that such shocks come from an imbalance of the foreign accounts which causes a change in the quantity of money that the Central Bank – under a fixed exchange rate – is not able to sterilise. We shall abstract from the possibility that shocks are either nominal or real and from the consequent search for the degree of indexation that minimises the real output variance over the business cycle in an environment where all agents are fully rational. We shall assume that the economy is only hit by nominal shocks, but that a given fraction \(\lambda\) of firms are non-responders to such shocks. See Gray (1976, 1978); Fischer (1977); Cuckierman (1980); Ball (1988); Ball, Cecchetti (1991).

\(^{11}\) Notice that (i) and (ii) support not only the *ex ante* but also the *ex post* desirability of stabilisation policies since output fluctuations *per se* imply welfare losses.

\(^{12}\) See, for instance, Boitani, Damiani (1999).
From the demand function and the production function one obtains the following expression for the profit of each firm:

\[ \pi_i = P \left( \frac{P}{P} \right)^{-\theta} M \frac{P}{P} - \left( \frac{P}{P} \right)^{\frac{\theta}{\alpha}} \left( \frac{M}{P} \right)^{\frac{1}{\alpha}} \]  

where \( h = \alpha + \theta(1 - \alpha) \)

By using the logarithmic transformation and rearranging one gets:

\[ \log P_i = \log \left( P \left( 1 - \frac{1 - \alpha}{h} \right) \right) + \frac{\alpha}{h} \left( \log w_i - \log \beta \right) + \frac{1 - \alpha}{h} \log M \]  

Notice that \( \frac{1 - \alpha}{h} \), say \( f \), is a real rigidity indicator, as it measures the responsiveness of \( \log P_i \) to aggregate demand, low \( f \) implies high real rigidity\(^{13} \), whilst \( 1 - \frac{1 - \alpha}{h} \), denoted henceforth as \( \rho \), is the degree of strategic complementarity.

Assume now the simple case that all the firms adopt a maximization strategy after the nominal shock, therefore \( \log P_i = \log P \). Inserting this condition in (7) the money multiplier then can be expressed as a function of \( f \) and \( \rho \):

\[ \frac{d \log P}{d \log M} = \frac{f}{(1 - \rho) \left[ 1 - \left( 1 - \frac{1 - \alpha}{h} \right) \right]} \]

which implies, given the symmetry properties of the model, that the general level of prices simply increase by a fraction \( \mu \).

These results are obtained under the hypothesis of a full indexation of the unemployment benefit to the price level, hence the real wage \( w_i \), which enters equation (7), is constant after the monetary shock. It is easy to show that the full indexation rule for the case in which all agents are fully rational is very simple. Consider the general indexation rule for the unemployment benefit:

\(^{13} \) See Ball, Romer (1990); Haltiwanger, Waldman (1991).
\[ \log B = \log b_o + k \log P \]  

where \( b_o \) is the value of the real benefit before the monetary shock and \( k \) is the parameter expressing the degree of benefit indexation.

The first order condition for profit maximisation must now be rewritten in order to introduce the real wage as a function of the real unemployment benefit (\( \log w = \log \gamma + \log B - \log P \)). By using the general indexation rule (9), the price charged by each maximising firm can be rewritten as follows:

\[
\log P_{im} = \left(1 - \frac{1}{h} + \frac{\alpha k}{h}\right) \log P + \frac{\alpha}{h} (\log \gamma + \log b_o - \log B) + \frac{(1-\alpha)}{h} \log M \quad (7')
\]

Notice that in case of full rationality all firms are maximizers (\( \log P = \log P_{im} \)), therefore from (7') one gets:

\[
d \log P = \frac{(1-\alpha)}{(1-\alpha k)} d \log M
\]

Under full indexation \( k = 1 \), hence the money multiplier is: \( \frac{d \log P}{d \log M} = \frac{(1-\alpha)}{(1-\alpha k)} = 1 \) and the long run equilibrium condition is restored. With full rationality indexing the nominal benefit to money is sufficient to keep constant the value of the real unemployment benefit. In our case, increasing the nominal benefit by a fraction \( \mu \) equal to the magnitude of the monetary shock (\( d \log B = d \log M \)), is sufficient to restore the initial values for the real variables.

In summary, if all agents are responders to a one-time monetary shock two main results are obtained: i) the rational behaviour assumption implies that firms correctly anticipate the magnitude of the monetary shock and adopt a simple price adjustment rule such as the following: \( \log P = \log P_o + d \log M \); ii) a policy designed to minimise output fluctuations entails the adoption of a simple indexation rule of the unemployment benefit in order to avoid that monetary disturbances have real effects.

IV. THE NEAR-RATIONAL BEHAVIOUR CASE

Akerlof Yellen (1985) proved that if there exists a small fraction of near-rational firms, a monetary shock is non neutral. When a long-run equilibrium (in which all agents maximise) is slightly
perturbed by a nominal shock, some monopolistically competitive firm may choose not to adjust its nominal price, i.e. some firm may choose not to maximise. The envelope theorem implies that firms, which are in principle able to adjust optimally, but which nevertheless do not respond to monetary shocks only incur second order profit losses from their non-maximising behaviour\textsuperscript{14}.

However, the macroeconomic consequences of what may be called \textit{near-rational} behaviour of individual firms are first order, even though near-rational firms are just a fraction of the firms’ population. The reason being that the non-adjusted prices of near-rational firms affect the price-adjustment behaviour of their fully rational competitors. Hence the aggregate price level changes to a smaller degree than the quantity of money, thereby triggering a real balance effect which leads to changes in real output\textsuperscript{15}.

In Akerlof and Yellen’s model the policy maker has no instrument capable of neutralising monetary shocks. We are interested in showing that when the government in principle does have such a policy instrument (i.e. an netralising indexation rule), the presence of some near-rational agents makes its use logically more complicated, as far greater information processing abilities on the part of the policy maker is required. Moreover, the introduction of some behavioural heterogeneity makes far more difficult for fully rational agents to form correct expectations about the price level.

In the following proposition it is shown that the full indexation rule for the unemployment benefit found above does not prevent money non-neutrality if there are \(\lambda\) near-rational firms, with high costs to collect and process information, which do not respond to small monetary shocks.

**PROPOSITION 1.** In the presence of a small fraction \((\lambda)\) of near-rational firms a full indexation rule such as \(\log B = \log b_n + k \log P\), with \(k = 1\), leads to \(\frac{dP}{dM} < 1\).

**PROOF** Following Akerlof, Yellen (1985), the general price level \(P\) can be expressed as a geometric mean of the price \(P_{in}\) charged by the \((1-\lambda)\) maximising firms and the price \(P_{in}\) charged by the \(\lambda\) non-maximising firms. In logs:

\[\log B = \log b_n + k \log P, \text{ with } k = 1, \text{ leads to } \frac{dP}{dM} < 1.\]

\textsuperscript{14} Notice that for a first order error in price adjustment to result in a second order profit loss the profit function of firms must be differentiable in its own price. A condition that does not apply under perfect competition. Hence an imperfectly competitive environment is necessary for the near-rationality argument to bite. (See Akerlof, Yellen (1985a, pp. 711-712; 1985b, pp. 826-827).

\textsuperscript{15} Similar, although not identical, results are found by Ball, Romer (1991) in a model featuring ‘small menu costs’.
\[ \log P = (1 - \lambda) \log P_{im} + \lambda \log P_{in} \quad (10) \]

Assume also that the non-maximising firms, after the shock \( M = M_o(1 + \mu) \), keep constant their price at the value \( P_0 \) set before the shock: \( \log P_{in} = \log P_0 \). Moreover suppose a full indexation rule \( \log B = \log b_o + \log P \), \((k=1)\) which guarantees that each monopoly union keeps constant the real wage \( w_i \).

For the rational firms, as already seen in section 2, the price strategy is given by equation (7).

Inserting in (7) \( \log P \) from (10), which applies in case of near rationality, is straightforward to get:

\[ \log P_{im} = (1 - \frac{1-\alpha}{h}) (\lambda \log P_0 + (1 - \lambda) \log P_{im}) + \frac{\alpha}{h} (\log w_i - \log \beta) + \frac{(1-\alpha)}{h} \log M. \]

Therefore the money impact on \( P_{im} \) is:

\[ \frac{d \log P_{im}}{d \log M} = \frac{(1-\alpha)}{\lambda h + (1-\lambda)(1-\alpha)} = \nu < 1 \text{ for } \lambda > 0 \quad (11) \]

Taking into account that \( \frac{d \log P}{d \log P_{im}} = (1 - \lambda) \), the money multiplier on the log of the general price level becomes:

\[ \frac{d \log P}{d \log M} = \frac{(1 - \lambda)(1-\alpha)}{\lambda h + (1-\lambda)(1-\alpha)} = \nu(1-\lambda) < 1 \text{ for } \lambda > 0 \quad (12) \]

q.e.d

By using the definition given in section 2 for the real rigidity indicator and the degree of strategic complementarity, the impact of the monetary shock can be easily expressed as a function of \( f, \rho, \) and \( \lambda \):

\[ \frac{d \log P_{im}}{d \log M} = \frac{f}{(1 - \rho(1-\lambda))} < 1 \text{ for } \lambda > 0 \quad (13) \]

\[ \frac{d \log P}{d \log M} = \frac{f (1 - \lambda)}{(1 - \rho(1-\lambda))} < 1 \text{ for } \lambda > 0 \quad (14) \]

The intuition behind this proposition, which extends Akerlof and Yellen's (1985b) original result to a slightly different model-economy, is that the presence of \( \lambda \) non-responders causes the degree of adjustment of \((1-\lambda)\) responders to differ from full adjustment: \( \frac{d \log P_{im}}{d \log M} < 1 \). Under strategic complementarity there is in fact an incentive for responders to imitate (at least up to a point)
the behaviour of non-responders. The overall degree of price adjustment is a weighted average of
the degree of adjustment of responders and non-responders: \( \frac{d \log P}{d \log M} \) will lie in between 0 and
\( \frac{d \log P_{\text{nc}}}{d \log M} < 1 \), hence it will surely be less than 1, implying money non-neutrality. The full indexation
rule is baffled by the presence of a fraction of non-responders; hence nominal shocks have real
effects.

A few remarks are in order (proofs are in the Appendix).

**REMARK A** The impact of a given amount of near-rational behaviour on the macroeconomic
outcome increases as the degree of strategic complementarity (\( p \)) increases (Haltiwanger and
Waldman, 1991, Proposition 4). As the degree of strategic complementarity grows, the synergistic
effects increase because of the bias of rational agents’ behaviour towards that of near-rational ones.
Conversely, given the degree of strategic complementarity, an increase in the share of non-responder
firms makes the divergence of the macroeconomic outcome from neutrality to increase.

**REMARK B** The nominal rigidity caused by the inertial behaviour of the near-rational firms
renders the monetary shock effective; the degree of effectiveness, however, is inversely related with
the degree of real flexibility (\( f \)). In terms of the traditional aggregate supply and demand framework,
it can be said that the non-responder behaviour of a given fraction of firms renders the aggregate
supply curve elastic instead of vertical. As real flexibility increases the elasticity of the aggregate
supply curve decreases, with consequent lower real effects of the shifts in the aggregate demand
curve generated by monetary shocks.

**REMARK C** The degree of real flexibility \( f \) depends negatively on \( \Theta \), i.e. the price elasticity of
demand, and positively on the degree of monopoly (\( 1/\Theta \)): real rigidity is lower the higher is the
degree of monopoly. Therefore the higher the monopoly power of firms the lower the degree of
strategic complementarity (\( p=1-f \)) and the lower the constraint on price increases. As a
consequence, the higher the degree of monopoly the lower the real effects of monetary shocks.

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In the presence of a fraction \( \lambda \) of near-rational firms, the full indexation rule seen above,
despite being ineffective at neutralising monetary shocks, becomes far more complex and information
demanding than in a world in which all agents are homogeneously rational. We are indeed able to prove the following:

**PROPOSITION 2.** *In the presence of a small fraction* \((\lambda)\) *of near-rational firms full indexation, i.e. the indexation which keeps the real wage constant, requires the government to know the structural parameters of the economy* \(\alpha\), \(\delta\) *and* \(\lambda\).

**PROOF** Assume a nominal shock of magnitude \(\mu\): \(d \log M = \log (1 + \mu)\). The money impact given by (12) is:

\[
d \log P = \nu (1 - \lambda) \ d \log M \quad (12')
\]

where, by using equation (11), \(\nu = \frac{(1 - \alpha)}{\lambda h + (1 - \lambda)(1 - \alpha)}\). To adopt a full indexation rule for the unemployment benefit, the policy maker has to be informed about the structural form of the model as it is possible to ascertain from the full indexation rule: \(\log B = \log b_0 + k \log P\) with \(k=1\). By using (12') one gets:

\[
d \log B = \nu (1 - \lambda) \ d \log M \quad (10'')
\]

where \(\nu\) is a function of \(\alpha\), \(\lambda\) and \(h\) which depends, on its turn, on \(\alpha\) and \(\delta\). *q.e.d.*

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As the full indexation rule does not guarantee the neutralisation of nominal shocks, one may wonder whether some endogenous perfect indexation rule may be found and whether such a rule would require more or less or the same information needed for the (sub-optimal) full indexation rule.

**PROPOSITION 3.** *In order to fully neutralise monetary shocks the government must over-index the unemployment benefit to a degree* \(k^* > 1\).

**PROOF** To obtain the degree of indexation of the unemployment benefit which guarantees that monetary disturbances have no real effects, a value \(k^*\) for the general benefit rule (9) is to be found such that

\[
\frac{d \log P}{d \log M} = 1.
\]

As \(k^* \neq 1\) implies that the real unemployment benefit \(\phi = B/P\) is no longer constant, the nominal benefit enters as an additional argument in the profit function of each \(i\)-th firm:

\[
\pi_i = \pi(P_{im}, P, M, B) \quad (15)
\]

The first order condition for the profit maximization, under the general indexation rule (9) for the unemployment benefit, is given by (7').
In case of near rationality the logarithm of the general price level is given by (10). Inserting (10) in (7') and assuming the starting condition \( \log P_0 = \log M_0 \)\(^{16} \) yields:

\[
\log P_{im} = \left(1 - \frac{1}{\alpha} \right) (\lambda \log P_0 + (1 - \lambda) \log P_{im}) + \frac{\alpha}{h} (\log \gamma + \log b_0 - \log \beta) + \frac{(1 - \alpha)}{h} \log M
\]

(16)

Solving for \( \log P_{im} \) leads to:

\[
\log P_{im} = a_1 \log P_0 + a_2 (\log \gamma + \log b_0 - \log \beta) + a_3 \log M
\]

(17)

where:

\[
\tau = \frac{\lambda(h-1+\alpha k)}{\lambda h + (1 - \lambda)(1 - \alpha k)} ; \quad a_3 = \frac{(1 - \alpha)}{\lambda h + (1 - \lambda)(1 - \alpha k)}
\]

Taking into account that \( \frac{d \log P}{d \log P_{im}} = (1 - \lambda) \), the money impact on the log of the general price level becomes:

\[
\frac{d \log P}{d \log M} = \nu (1 - \lambda) = \frac{(1 - \alpha)(1 - \lambda)}{\lambda h + (1 - \lambda)(1 - \alpha k)}
\]

(18)

The value of \( K \) which satisfies the condition \( \frac{d \log P}{d \log M} = 1 \) is therefore \( k^* = l + \frac{\lambda h}{(1 - \lambda) \alpha} > 1. \),

q.e.d.

Less than perfect rationality of a fraction of firms makes things more difficult for the policy maker, at least in principle. If all firms are perfectly rational responders, and the policy maker knows they are, all the information he needs in order to determine the full indexation rule (which is also neutralising in that case) is the magnitude of the shock. Just a little heterogeneity is enough for the amount of information needed to increase. It is not enough to know that “some” firms are less-than-rational. The precise share of non-responders \( \lambda \) must also be known, besides such structural parameters as the price elasticity of demand (\( \varphi \)) and the inverse of the output elasticity of employment (\( \varphi \))\(^{17} \).

\(^{16} \) If \( k = 1 \), the starting condition \( \log P_0 = \log M_0 \) and the symmetrical equilibrium condition \( \log P_{im} = \log P \) are verified from (7') for \( (\log \gamma + \log b_0 - \log \beta) = 0 \). Therefore, the price equation for the maximising firm can be written as (16).

\(^{17} \) Notice that the number of parameters to be known is kept to a minimum by the simplicity of the model employed in the present paper. In more complete models the number of structural parameters the policy maker needs information about obviously increases, making things even more difficult.
It is not just the quantity of information that changes but its quality as well. Under the assumption of homogeneous full rationality the policy maker only needs to know that the economy reacts to nominal shocks as the (neo)classical theory predicts. Under heterogeneity in order to predict the aggregate behaviour of the economy one needs to know the structural model. Thus, with homogeneous full rationality the policy maker may rely on observations of aggregate behaviour, under heterogeneity additional information is required even if the nominal shock repeats itself at every time and $\lambda$ is stable over time. Things become even more complicated if the magnitude of the nominal shock fluctuates and/or if $\lambda$ changes over time. Under full indexation case it is sufficient that the policy maker is able to collect information on two structural parameters out of three and does actually know equation (12). If, for instance, the government has information about $\alpha$ and $\vartheta$ from industry data, by observing $\frac{d\log P}{d\log M} = \Delta$, from equation (12) it may calculate: $\lambda = \frac{\Delta(1-\alpha) - 1 + \alpha}{(1-\alpha)|1 + \Delta(1-\vartheta)| - u\Delta}$. Under full indexation $k=1$ by definition, whilst perfect indexation requires $k$ to be endogenously determined: $k^*$ is obtained from equation (18) by imposing $\frac{d\log P}{d\log M} = 1$. In the model the sole source of information about $\lambda$ is again equation (18), provided $\alpha$ and $\vartheta$ are known in some way. It should also be noticed that the policy maker cannot observe $\frac{d\log P}{d\log M} = 1$ unless $k=k^*$, which needs knowledge of $\lambda$. Additional information coming from outside the model is needed to implement a neutralising policy.

V. PARADOXES OF RATIONALITY

Given that the full indexation rule $\log B = \log b_o + k \log P$, with $k=1$, is not able to neutralise monetary shocks, that the neutralising indexation rule requires over-indexation and non constant real wages, besides both rules being informationally demanding, one may wonder whether a simple rule such as indexing the nominal benefit to the quantity of money would perform well in limiting the real impact of monetary shocks, i.e. whether a less-than-well informed maker wouldn’t make great harm to the macro-economy. We are in fact able to prove the following:

**PROPOSITION 4.** If the government ignores the presence of $\lambda$ non-responders and adopts a simple indexation rule such as $\log B = \log B_0 + \log(1+\mu)$, the real impact of monetary shocks is lower than under full indexation.

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18 It should perhaps be noticed that in the full indexation case it is sufficient that the policy maker is able to collect information on two structural parameters out of three and does actually know equation (12). If, for instance, the government has information about $\alpha$ and $\vartheta$ from industry data, by observing $\frac{d\log P}{d\log M} = \Delta$, from equation (12) it may calculate: $\lambda = \frac{\Delta(1-\alpha) - 1 + \alpha}{(1-\alpha)|1 + \Delta(1-\vartheta)| - u\Delta}$. Under full indexation $k=1$ by definition, whilst perfect indexation requires $k$ to be endogenously determined: $k^*$ is obtained from equation (18) by imposing $\frac{d\log P}{d\log M} = 1$. In the model the sole source of information about $\lambda$ is again equation (18), provided $\alpha$ and $\vartheta$ are known in some way. It should also be noticed that the policy maker cannot observe $\frac{d\log P}{d\log M} = 1$ unless $k=k^*$, which needs knowledge of $\lambda$. Additional information coming from outside the model is needed to implement a neutralising policy.
**PROOF** Consider again the effect of a monetary disturbance in an environment where the indexation rule adopted to fix the unemployment benefit does not guarantee the stability of the real benefit. However in the case at hand the policy maker adjusts the nominal benefit according to the magnitude of the monetary shock. Therefore the impact of the nominal disturbance is partially indirect, thorough the effect on $B$.

Rewrite for convenience the price equation for the rational firms:

$$\log P_{im} = \log P + \frac{\alpha}{h} \left( \log \gamma + \log B - \log P - \log \beta \right) + \frac{(1-\alpha)}{h} (\log M - \log P) \quad (7')$$

As the full indexation of the unemployment benefit to the money level keeps constant the ratio $\frac{B}{M}$, we have an indexation rule such as: $B = g_0 \cdot M$, where $g_0$ is the initial value of $\frac{B}{M}$ before the nominal shock. Then, expressing $\log B$ as $\log B = \log g_0 + \log M$ and replacing it into the price equation, one gets:

$$\log P_{im} = \log P + \frac{\alpha}{h} \left( \log \gamma + \log g_0 + \log M - \log P - \log \beta \right) + \frac{(1-\alpha)}{h} (\log M - \log P)$$

For $\log P_0 = \log M_0$ the price equation can be written as follows\(^{19}\):

$$\log P_{im} = (1 - \frac{1}{h}) \log P + \frac{1}{h} \log M$$

Taking into account that $\log P = (1 - \lambda) \log P_{im} + \lambda \log P_0$ yields:

$$\log P_{im} = \tau'' \log P_0 + \nu'' \log M \quad (19)$$

where: $\tau'' = \frac{\lambda(h-1)}{\lambda h + (1-\lambda)}$; $\nu'' = \frac{1}{\lambda h + (1-\lambda)}$

As $\frac{d \log P}{d \log P_{im}} = (1 - \lambda)$, then the money multiplier is:

$$\frac{d \log P}{d \log M} = (1 - \lambda) \cdot \nu'' = \frac{(1 - \lambda)}{\lambda h + (1 - \lambda)} \quad (20)$$

As $\alpha < 1$, it holds true:

\(^{19}\) As already shown, for $\log P_0 = \log M_0$ we obtain the condition $\left( \log \gamma + \log b_0 - \log \beta \right) = 0$. Under the same condition $\log \left( \frac{B_{u0}}{P_0} \right) = \log b_0 = \log \left( \frac{B_{u0}}{M_0} \right) = \log g_0$. Therefore $\left( \log \gamma + \log g_0 - \log \beta \right) = 0$.
\[
\frac{1-\lambda}{\lambda h + (1-\lambda)} > \frac{(1-\alpha)(1-\lambda)}{\lambda h + (1-\lambda)(1-\alpha)} \quad q.e.d.
\]

Paradoxically, the ignorance of the government reduces the impact of a monetary shock on the real variables, as it can be verified comparing the money multiplier (20) with that obtained in the full indexation case (12). Intuitively, by neglecting the existence of non-responder firms and indexing the unemployment benefit to the monetary shock, the policy maker causes such a benefit to increase in real terms and this partly compensates the expansionary effect of the nominal shock.

***

Another interesting result emerges if one assumes that responder firms have perfect foresight on exogeneous shocks but ignore the presence of non-responders. In other words, the \((1-\lambda)\) responder firms are rational in the sense that they adjust their prices in order to maximise profits \(\text{ex ante}\), but they are affected by some “near-rationality” in the forecasting process, since they attribute their maximising behaviour also to non-maximising firms. The result is that a non-perfectly rational forecasting process on the part of responder firms leads to more stability of the output level. In order to illustrate this result we shall state and prove the following:

**Lemma**: In the presence of \(\lambda\) non responder firms and \((1-\lambda)\) rational, but non-perfectly informed, responders both the price level and the output level are independent of the indexation rule followed by the policy maker.

**Proof**: As responder firms ignore the presence of the \(\lambda\) non responders, their best expectation (given their information) of the price level change after the observed shock \(d \log P^e\) can be calculated from the money multiplier for the full rationality case, which, as shown in section 3, is given by the following expression: \(d \log P^e = \frac{(1-\alpha k)}{(1-\infty)} d \log M\). Moreover, responder firms know that under full rationality the simple indexation rule \(k=1\) is sufficient to keep the real variables constant, hence their expectations of the price level and the unemployment benefit are formed assuming that \(k^e = 1\). Therefore \(\frac{d \log P^e}{d \log M} = 1\) and \(\frac{d \log B^e}{d \log P^e} = \frac{d \log B^e}{d \log M} = 1\); which implies that the expected real wage \(w^e\) will be unchanged.

The price-adjustment maximising strategy (from equation 7) under this wage and price expectations then becomes:
\[ d \log P_{im} = d \log P^e (1 - \frac{1 - \alpha}{h}) + \frac{(1 - \alpha)}{h} d \log M. \]

After substituting for \( d \log P^e \), one obtains:

\[ d \log P_{im} = d \log M (1 - \frac{1 - \alpha}{h}) + \frac{(1 - \alpha)}{h} d \log M \]

which gives:

\[ \frac{d \log P_{im}}{d \log M} = 1 \]

The optimal price decision of a maximising firm that believes to live in a world of full rationality is therefore to adjust its price according to the magnitude of the nominal shock. As the non responders agents are keeping constant their prices, the actual price level will be:

\[ P = (P_0 (1 + \mu))^{(1 - \lambda)} P_0^\lambda = P_0 (1 + \mu)^{(1 - \lambda)} \quad q.e.d^{20} \]

It is now easy to prove the following:

**PROPOSITION 5** The degree of non-neutrality is lower when responder firms are less-than-well informed than when they have perfect knowledge.

**PROOF** The price charged by maximising firms is set according to the following rule:

\[ \log P_{im} = \log P_0 + d \log M \]

If the \( \lambda \) non-responder firms set their prices at the level before the shock, \( \log P_{im} = \log P_0 \), the aggregate price level is:

\[ \log P = \log P_0 + (1 - \lambda) d \log M \]

Comparing this result with that obtained under the full rationality hypothesis, it is easy to get:

\[ \log P_0 + (1 - \lambda) d \log M > \log P_0 + \nu(1 - \lambda) d \log M \quad \text{as} \quad \nu < 1 \quad q.e.d. \]

The macroeconomic implication of this result is that monetary shocks cause more significant fluctuations in real variables the more correct are the conjectures of maximising firms. By ignoring the presence of non-responders, responder firms do not perceive the incentive to “match” the near-rational behaviour. Therefore strategic complementarity is less powerful. A learning process that allows rational firms to take into account the presence of non maximising firms would cause the model economy to exhibit amplified fluctuations in real variables.

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20 In the long run, different indexation rules followed by the policy maker are indeed effective, as different indexation rules affect profits of both responder and non-responder firms and trigger price and quantity changes.
In order to compare the cases examined here and in section 4, according to the degree of effectiveness of monetary shocks, TABLE 1 summarises the results found so far. Different cases are ranked in terms of their stability properties in the face of nominal disturbances.

<table>
<thead>
<tr>
<th>Case Description</th>
<th>Price effect</th>
<th>Output effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Neutralising indexation</td>
<td>$d \log P = d \log M$</td>
<td>$d \log Y = 0$</td>
</tr>
<tr>
<td>2) Less-than-well informed responders</td>
<td>$d \log P = (1-\lambda)d \log M$ $(1-\lambda) &lt; 1$</td>
<td>$d \log Y = \lambda d \log M$</td>
</tr>
<tr>
<td>3) Less-than-well informed government</td>
<td>$d \log P = \nu(1-\lambda)d \log M$ $\nu(1-\lambda) = \frac{(1-\lambda)}{\lambda h + (1-\lambda)} &lt; (1-\lambda)$</td>
<td>$d \log Y = (1-\nu(1-\lambda))d \log M$</td>
</tr>
<tr>
<td>4) Full indexation</td>
<td>$d \log P = \nu (1-\lambda)d \log M$ $\nu = \frac{(1-\infty)}{\lambda h + (1-\lambda)(1-\infty)} &lt; \nu^*$</td>
<td>$d \log Y = (1-\nu(1-\lambda))d \log M$</td>
</tr>
</tbody>
</table>

In an environment featuring heterogeneous behaviour, as that considered in the present paper, money non-neutrality is obtained even if every agent has correct expectations on monetary disturbances, unless the policy maker is able to find a fully neutralising indexation rule. If such a neutralising rule cannot be found, it turns out that more accurate are the anticipations of responder firms on non-responders’ behaviour, the higher is the degree of policy effectiveness. As far as the output stabilisation goal is concerned, rule-of-thumb behaviour leads to better results than (unsuccessful) attempts at neutralising the presence of near-rational firms.

As a final question, one may wonder whether the adoption of simple behavioural rules may be convenient in terms of individual objectives. Akerlof and Yellen (1985a), showed that when a long-run equilibrium is slightly perturbed by a nominal shock, some monopolistically competitive firm may choose not to adjust their nominal price, i.e. some firm may choose not to maximise. The envelope theorem implies that firms, which are in principle able to adjust optimally, but which nevertheless do not respond to monetary shocks only incur second order profit losses from their non-maximising behaviour.

Here we intend to obtain additional insights on the profit outcomes of the maximising agents. The $(1-\lambda)$ firms who choose to adjust their nominal prices, may have accurate knowledge of the $\lambda$.
firms, or simply ignore their inertial behaviour. To evaluate gains or losses that the (1-\(\lambda\)) maximiser firms may obtain in the various cases examined so far, some numerical simulations have been performed. The various cases examined and ranked in TABLE 1 according to a macroeconomic criterion have been compared in terms of profitability (TABLE 2). The full rationality of all agents (\(\lambda=0\)) has been selected as the benchmark situation and the profit outcomes that the i.th maximiser firm achieves are calculated as percentage differences with respect to the full rationality case.

The numerical simulations reported in TABLE 2 have been performed for different values of \(\lambda\), a nominal shock of +3%, \(\Theta=5\) and the initial condition \(\frac{M_o}{P_o} = 1\).

<table>
<thead>
<tr>
<th></th>
<th>(\lambda=0.25)</th>
<th>(\lambda=0.5)</th>
<th>(\lambda=0.75)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Neutralising indexation</td>
<td>-3.86%</td>
<td>-11.15%</td>
<td>-29.86%</td>
</tr>
<tr>
<td>2) Less than well informed responders</td>
<td>+0.36%</td>
<td>+0.62%</td>
<td>+0.78%</td>
</tr>
<tr>
<td>3) Less than well informed government</td>
<td>-1.92%</td>
<td>-3.01%</td>
<td>-3.60%</td>
</tr>
<tr>
<td>4) Full indexation</td>
<td>-3.06%</td>
<td>-3.63%</td>
<td>-3.82%</td>
</tr>
</tbody>
</table>

The most surprising result is shown in the second row. It comes out that if a maximising (responder) firm is not well informed, hence adopts not fully consistent model expectations, not only does better than when is perfectly informed but does also better than when all firms are perfectly rational. The presence of some near rational firm – provided they are ignored – turns out to be good for rational firms.

The intuition is as follows. A less than well informed responder, failing to anticipate the inertial behaviour of non responder firms, sets a higher relative price, hence it will experience a contraction of sales. However, this negative impact on revenue is mitigated by the fact that there will be a rise in aggregate demand, as the presence of \(\lambda\) non responders implies \(d \log P < d \log M\). Moreover, on the cost side, as the LEMMA above shows, there will be stability of the unit labour cost, as the government cannot but adopt a full-indexation rule.
It should also be emphasised that it is not because of inaccurate predictions that rational agents obtain additional returns (with respect to a perfect foresight situation). It is because that, when rational agents have limited information, there is no scope for government intervention aimed at neutralising nominal shocks; hence the over-indexation rule of the unemployment benefit is simply not adopted. In such circumstances, rational agents achieve some extra gains as they take advantage of the positive aggregate demand externality, due to the inertial behaviour of non responder agents. Moreover, losses caused by higher labour costs are in this case avoided, since the real unemployment benefit can be held constant.

The worst results are achieved when the maximiser firm has perfect foresight and the government adopts a neutralising indexation rule of the unemployment benefit to prices, as shown in the first row of TABLE 2. In this situation, the increase of the maximiser relative price is not counterbalanced by the increase in the aggregate activity, as the government has to stabilise \( \frac{M}{P} \) in order to achieve the macroeconomic stability of output. Moreover, the over-indexation rule \( (k^*>1) \) of the unemployment benefit, which guarantees the macroeconomic target, implies a higher labour cost. As \( \lambda \) increases, the degree \( (k) \) of indexation of the unemployment benefit increases and the maximiser profit losses are magnified, as it is possible to check by comparing the results obtained in the first row for different values of \( \lambda \).

Summing up, while the government may contain welfare losses due to aggregate fluctuations by acting as a well informed policy maker, the profit outcomes results suggest that near-rationality reduces the incentive for individual maximisers to form accurate predictions on the degree of non responsiveness in the model economy, making more plausible, the adoption of more profitable, albeit less sophisticated, strategies and reducing the incentive for a revision of expectations and decision making procedures. Therefore, near rationality not only causes a “bias” in the action of responders towards that of non responders, but it also may induce deviations from fully rational expectations rules.

---

21 The crucial role of the degree of near rationality is apparent in all the examined cases: as the number of non responder firms increases the profit deviations from the full rationality case are more relevant.
VI. CONCLUSIONS

Standard economic analysis is built on the assumption of homogeneous agents, an assumption which allows one using models based on a representative agent. Some relevant macroeconomic results, such as the money-neutrality proposition, are obtained by making the representative agent assumption. Several papers have shown that those results are not robust as, for instance, an however small behavioural heterogeneity leads to non-neutrality. In this paper we moved a step further and explored the implications of heterogeneous behaviour on the ability of policy makers to implement neutralising policies in the face of nominal shocks (for the sake of simplicity real shocks were ruled out).

We began by showing that, when all agents (firms) are homogeneously rational, that is they all respond optimally to nominal shocks, the neutralising goal may be achieved by indexing the pivot nominal variable (the unemployment benefit in our model) to the magnitude of the nominal shock. Such an indexation rule is neutralising yet simple, as it only requires that the policy maker knows that everybody is rational and that the economy behaves as Milton Friedman would predict.

As soon as a small fraction of near-rational (non-responder) firms is introduced things change substantially. First, we showed that the simple rule is no longer neutralising, because non-responder firms cause the price level to change to a lesser proportion than the nominal shock. Second, we showed that a standard full indexation rule is also non-neutralising, although it is very information demanding. The rule now requires that the policy maker knows the whole set of structural parameters of the model-economy, which require that additional information is collected and processed. Third, the neutralising rule turns out to be an over-indexation rule, which does not guarantee the constancy of the real wage and is even more information demanding than the full indexation rule. Fourth, we showed that, under heterogeneity, if rational agents and policy makers adopt simple rule-of-thumb behaviour the real impact of nominal shocks may be smaller than if sophisticated (though non-optimal) rules are attempted at. By numerical simulation we were also able to show that the presence of near rational agents may significantly weaken and even reverse the individual incentive of rational agents to adopt sophisticated behavioural rules.

The propositions in the previous sections assume that rational responder firms have perfect foresight. Things become even more complicated as soon as uncertainty is introduced and perfect
foresight is replaced by some form of expectations-formation rule. A proper treatment of this issue would require a fully specified dynamic model. Hereafter only a few tentative observations are advanced.

As underlined by Phelps and Frydman (1983), in order to forecast the values of endogenous variables, each rational firm has to form expectations on the economy-wide average opinion on these variables. In the RE framework, every agent solves this difficult task by assuming that other agents share the same expectations; hence the RE approach ‘entails perceived and actual unanimity of beliefs across all agents ...’ (p.7). In this sense, it can be said that the RE approach is inherently dominated by the homogeneity hypothesis.

Introducing behavioural heterogeneity has far-reaching implications. First, rational agents who know that some other agents do not fully respond to monetary shocks must assign to these near-rational agents a reply function which is necessarily different from their own reply function. Second, under heterogeneity, each rational agent has to forecast not only how other rational agents forecast exogenous shocks, but also how they perceive the behaviour of non-responder agents, even if all agents - either rational or near-rational - share the same conjectures on exogenous shocks. Consequently, the difficulty stressed by Phelps and Frydman (1983) is magnified. Under homogeneity it seems legitimate for rational agents to project their own views on to other agents and thus to predict the economy-wide average opinion. With heterogeneous agents, on the other hand, there is no longer a single theoretical model of price adjustment behaviour, which may guide the individual price-setter to find out the economy-wide average opinion.

In this context, non-neutrality may arise not only because of the inertial behaviour of near-rational firms. It can also come about because rational firms fail to anticipate the extent of non responsiveness in the model economy and/or they do not correctly perceive the average opinion about this degree of non responsiveness. As the heterogeneity of price expectations is the natural consequence of behavioural heterogeneity, it seems to apply a fortiori what Pesaran (1987) says about heterogeneous information: ‘under heterogeneous information decision making will be subject to behavioural uncertainty, and a rigorous derivation of the rational expectations models from

22 Moreover the problems raised by imperfect competition for agents' forecasting tasks should be examined. See Rankin (1992,1995, 1997).
principles of economic optimisation generally will not be possible’ (pp. 70-71). Whether a convergent learning process can be found under persistent heterogeneity is matter for further research.

**APPENDIX**

This appendix provides proofs of Remarks a, b and c in section 4.

**REMARK A**

Let us call \( P_m^* \) the general level of prices in the case all agents are rational. The money multiplier is then by definition equal to the one given in equation (8):

\[
\frac{dP_m^*}{dM} = \frac{f}{(1-\rho)} \frac{P_0}{M} \quad (A.1)
\]

whilst \( \log P \) is the log of the general level of prices when a fraction \( \lambda \) of firms are non responders, so that the money multiplier is given by equation (14):

\[
\frac{d\log P}{d\log M} = \frac{f(1-\lambda)}{[1-\rho(1-\lambda)]} < 1 \quad (A.2)
\]

Following Haltiwanger and Waldman (1991) one may subtract (A.2) from (A.1) to get:

\[
0 < -\frac{d(P_m^* - P)}{dM} = \frac{f \lambda}{(1-\rho)[1-\rho(1-\lambda)]} \quad \text{for } \lambda > 0 \quad (A.3)
\]

The proof of Remark a is now just obtained by differentiation:

\[
\frac{\partial d}{\partial \lambda} = \frac{h^2 \lambda (2(1-\alpha)(1-\lambda) + h \lambda)}{(1-\alpha)(1-\alpha - \lambda + \alpha \lambda + h \lambda)^2} > 0 \quad (A.4)
\]

\[
\frac{\partial d}{\partial \rho} = \frac{f}{(1-\rho + \rho \lambda)^2} > 0 \quad (A.5)
\]

**REMARK B**

As \( \rho = (1-f)^{23} \), after substitution in \( d \), by differentiation one gets:

\[
\frac{\partial d}{\partial f} = \frac{\lambda}{(1-\rho)(1-\rho(1-\lambda))} > 0 \quad (A.6)
\]

**REMARK C**

Write \( \frac{1}{\Omega} \equiv \omega \) the degree of monopoly power, than by differentiating \( f \) with respect to \( \omega \):

\[
\frac{\partial f}{\partial \omega} = \frac{(1-\alpha)^2}{[(\alpha + \omega^{-1}(1-\alpha))\omega]} > 0 \quad (A.7)
\]

Combining (A.6) and (A.7) one immediately obtains:

---

23 The negative relation between strategic complementarity and real flexibility has been shown, in a more general framework, by Alvi (1993).
\[
\frac{\partial d}{\partial \omega} = \frac{\partial d}{\partial f} \frac{\partial f}{\partial \omega} < 0 \tag{A.8}
\]

REFERENCES


