Process innovation and licensing

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Abstract
We consider technology transfer from a leader, who has the most productive technology, to a follower under licensing by means of a two-part tariff (fixed fee and a royalty). It is shown that the optimal contract includes a royalty. The royalty rate exceeds the difference in costs as it acts as an implicit collusive device. Hence, social welfare may be reduced as a consequence of the licensing contract.
These results do not hold in the Cournot case.
Then we consider a monopolist with no competitors who creates one intentionally and licenses a firm (a follower) able to produce goods by a cost-reducing technology; this case is dual to the above examined.
Finally, we analyze procurement and find that it dominates licensing when output is not contractible.

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Introduction

We set up a model with an industry composed of two firms: a leader owning a cost-reducing innovation, and a follower producing homogeneous goods (section 1).

We consider the possibility of technology transfer from the leader that has (and uses) the most productive technology to the follower under licensing by means of a two-part tariff (fixed fee and a royalty).

The interaction between leader and follower is formally analyzed as a non-cooperative two-stage game. In the first stage, the leader offers a (two-part tariff) licensing contract, the follower then chooses whether to accept it or reject it. In the second stage, the firms engage in a quantity competition game, the Stackelberg equilibrium of which determines their individual profits. An alternative would be to assume the Nash bargaining solution where the agreement reached in the first stage is the one which maximizes the product of rents (section 2).

Then we consider a monopolist with no competitors who creates one intentionally and licenses a firm (a follower) able to produce goods by a cost-reducing technology; this case is dual to the above analyzed (section 3).

Finally, if the output is not verifiable, a procurement contract leading to a Stackelberg model is considered and compared to licensing (section 4).


All the cases discussed in the paper lead to a Stackelberg framework and the paper provides three contributions to existing literature.

The main contribution is that the optimal licensing contract includes a royalty and the optimal royalty rate (the leader charges) exceeds the difference in costs counter-intuitively. The royalty acts as an implicit collusive device. In fact, the leader does not rise its production and in order to restrain the follower’s output, the royalty is greater than the difference in costs. The output price then rises after licensing.

Social welfare measured as the sum of consumer surplus and firms’ profits may therefore shrink as a consequence of the licensing agreement.

These results do not hold in the Cournot case.
Secondly, a monopolist owning a high cost technology finds it profitable to use its own technology and to license the production of homogenous goods to a firm able to produce them by a cost-reducing technology.

The last contribution shows that procurement can be re-interpreted as a Stackelberg model and its comparison with licensing gives some insights on different types of subcontracting.

1. Cost reduction and profits

Consider a Stackelberg quantity competition model with two firms, a leader (1) and a follower (2), producing homogeneous goods.

The demand function is linear:

\[ p = 1 - q \]

where \( p \) is the price of the homogenous goods and \( q \) is the output of the goods.

Both firms produce at constant unit production cost \( c_1 \), and \( c_2 \) \((0 < c_i < 1)\).

Equilibrium outputs, price and profits are:

- \( q_1 = \frac{(1 - 2c_1 + c_2)}{2} \) \( \quad (1) \)
- \( q_2 = \frac{(1 + 2c_1 - 3c_2)}{4} \) \( \quad (2) \)
- \( p = \frac{(1 + 2c_1 + c_2)}{4} \) \( \quad (3) \)
- \( \Pi_1 = \frac{(1 - 2c_1 + c_2)^2}{8} \) \( \quad (4) \)
- \( \Pi_2 = \frac{[(1 + 2c_1 - 3c_2)/4]^2}{8} \) \( \quad (5) \).

Now, let’s consider process innovation by firm 1 that lowers its unit cost by the amount \( \varepsilon \) and, for the sake of convenience, impose \( c_1 = c_2 = c \). Thus, the unit production cost for firm 1 is \( c - \varepsilon \), and for firm 2 is \( c \).

The follower stays active (i.e., \( q_2 \) is positive) provided that

\[ \Pi_2 = \Pi_1 \]
\[(1- c)/2 > \varepsilon.\]

If this inequality is reversed, the leader becomes a monopolist. The above condition separates, in absolute and relative terms, drastic \((\leq)\), when the more efficient firm sets its monopoly, from non drastic \((>\)\) differences in costs, when the market is made by the two firms.

In the non drastic case, the new outputs, price and profits are:

\[
\begin{align*}
q_1 &= \frac{(1 - c + 2\varepsilon)}{2} \\
q_2 &= \frac{(1 - c - 2\varepsilon)}{4} \\
p &= \frac{(1 + 3c - 2\varepsilon)}{4} \\
\Pi_1 &= \frac{(1 - c + 2\varepsilon)^2}{8} \\
\Pi_2 &= \left[\frac{(1 - c - 2\varepsilon)}{4}\right]^2.
\end{align*}
\]

For drastic differences in costs and for \(c + \varepsilon > 1\), monopoly output, price and profits are given by:

\[
\begin{align*}
q_1 &= q_M = \frac{(1 - c + \varepsilon)}{2}, \\
p_M &= \frac{(1 + c - \varepsilon)}{2} \\
\Pi_M &= \left[\frac{(1 - c + \varepsilon)}{2}\right]^2.
\end{align*}
\]

If \(c + \varepsilon < 1\), when \(q_1 = \frac{(1 - c + \varepsilon)}{2}\), then \(q_2 = \frac{(1 - c - \varepsilon)}{4} > 0\). So firm 1 must increase its output in order to get \(q_2 = 0\):

\[
\begin{align*}
q_1 &= q_{M1} = (1 - c), \\
p_{M1} &= c \\
\Pi_{M1} &= \varepsilon (1 - c).
\end{align*}
\]

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4. Most results hold for quadratic or other non linear cost functions.

5. It is an adaptation of the drastic and non drastic innovation differences discussed by K. Arrow (1962). A drastic innovation arises in case the monopoly price by means of the new technology does not exceed the competitive price under the old technology (M. Kamien and Y. Tauman, 1986 p.472).
2. The incentive to license: fee versus royalty licensing

We consider the possibility of technology transfer from the leader to the follower under licensing, ignoring information problems. Even if the leader has production capabilities, he may still license a rival. Licensing can occur only if it rises the leader’s profits and if the follower’s net profits remain, at least, at the level reached before licensing. For simplicity, we assume that when the follower is indifferent between accepting the leader’s licensing offer and rejecting it, it chooses to accept the offer (i.e. it licenses from the leader).

The leader (licensor) allows the follower (licensee) to use its cost-reducing innovation by means of a two-part tariff. The licensee pays a fixed fee to access the innovation and then a variable fee or a royalty per produced goods unit (or a sales percentage). The output is verifiable even though this is not necessary in the fixed fee case.

In the next paragraphs we consider a fixed fee and a two-part tariff. We prove that the optimal licensing contract includes a royalty and we compare fixed fee to royalty licensing and prove that it is the latter that maximizes the leader’s profits. The interaction between leader and follower is formally analyzed as a non-cooperative two-stage game. In the first stage the leader offers a (two-part tariff) licensing contract, and the follower chooses whether to accept it or reject it. An alternative which is also analyzed, is the Nash bargaining solution where the agreement maximizes the product of the rents. In the second stage firms engage in a quantity competition game, the Stackelberg equilibrium of which determines their individual profits.

2.1 Fixed fee licensing

In this subsection we consider licensing by means of a fixed fee only. Under this method the leader licenses its new technology to the follower at a fixed fee F which entitles it to use the new technology to produce as many as units it wishes.

Imposing \( c_1 = c_2 = c - \epsilon \) into equations (1-5), the firm’s equilibrium outputs, price and profits are given by (the subscript F denotes fee licensing):

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6 Many authors allow either fixed fees or royalties only. M.Katz and C.Shapiro (1985) argue that the licensee's output may not be observable and that the fixed fee is a good approximation of the real world, whereas in N.Gallini and R. Winter (1985) only royalties are considered. For additional reasons to use the royalty payment see M.Katz and C.Shapiro (1986 p.588).

7 A.Mukherjee (2001) analyses the fixed fee mechanism in some circumstances where the internal patentee behaves as a Stackelberg leader.
\[ q_{1F} = (1 - c + \varepsilon)/2 \]
\[ q_{2F} = (1 - c + \varepsilon)/4 \]
\[ p_F = (1 + 3(c - \varepsilon))/4 \]
\[ \Pi_{1F} = (1 - c + \varepsilon)^2/8 \]
\[ \Pi_{2F} = [(1 - c + \varepsilon)/4]^2 \]

With non drastic differences in costs the maximum fee the leader can charge is:

\[ F = \Pi_{2F} - \Pi_2 = 3 \varepsilon (2 - 2c - \varepsilon)/16 \]

and the leader’s total income (profits plus fixed fee) is larger than the profits in the no-licensing case, i.e. \( \Pi_{1F} + F > \Pi_1 \) iff \( 2(1-c)/9 > \varepsilon \). The leader will license its new technology if \( 2(1-c)/9 > \varepsilon \), and will not if this does not hold. Such condition is more restrictive than the one required by non drastic differences in costs. It means that total profits after licensing are larger than those in the no-licensing case.

With drastic differences in costs the maximum fee the leader can charge is:

\[ F = \Pi_{2F} - \Pi_2 (= 0) = [(1 - c + \varepsilon)/4]^2 \]

and the leader’s total income (profits plus fixed fee) is always lower than the profits in the no-licensing case, for \( \varepsilon > (1-c) \):

\[ \Pi_{1F} + F - \Pi_M = -[(1 - c + \varepsilon)/4]^2 \]

and for \( \varepsilon < (1-c) \):

\[ \Pi_{1F} + F - \Pi_{M1} = -3[(1 - c + \varepsilon)/4^2\varepsilon (1 - c)] \]

i.e. the differences between total profits in the Stackelberg and in the monopoly case. Hence, under the fixed fee licensing method the leader will not license its new technology and will become a monopoly with drastic differences in costs.
2.2 Royalty licensing

We now examine two-part tariff licensing contracts. The leader licenses the use of the innovation in exchange for a fee, F, and a royalty, r, per unit of output.

For any given r, the follower’s per unit cost of production under licensing is \((r + c - \varepsilon)\). Solving for the reaction functions of the leader and the follower yields, for any given royalty set at the first stage, the market-competition equilibrium outputs, price and profits (the subscript \(R\) denotes royalty licensing):

\[
q_{1R} = \frac{(1- c + \varepsilon)}{2}\\
q_{2R} = \frac{((1- c + \varepsilon)/4) - r/2}{1/4}\\
p_R = \frac{((1+ 3c - 3\varepsilon)/4) + r/2}{1/4}\\
\Pi_{1R} = \frac{((1 -c + \varepsilon)^2/8} + [r(1 - c + \varepsilon)/4]\\
\Pi_{2R} = \frac{((1 - c + \varepsilon)/4 - r/2)^2}{1/4}.
\]

Note that by moving from Cournot to Stackelberg competition, one adds an important effect to the market competition stage: when choosing quantity, the leader considers not only its effect on its own market profits, but also on follower’s quantities, that determine its royalty revenues. \(^8\)

At the first stage, the leader will choose \((r, F)\) in order to maximize its profits subject to the follower’s participation constraint, that is:

Max \(\Pi_{1RT} = \Pi_{1R} + r q_{2R} + F\)

s.t. \(\Pi_{2R} - F \geq \Pi_2\).

At the optimum:

\(F = - \Pi_2 = - [(1 - c -2\varepsilon)/4]^2\) and \(r = [(1 - c + \varepsilon)/2]\). Which implies: \(q_{1R} = q_M\) (monopoly output), \(q_{2R} = 0\). That is, if the only constraint to the leader’s maximization problem is the follower’s participation constraint, then the optimum two-part tariff licensing contract implements the monopoly outcome and the leader’s profits attain the first-best level. However, the first-best optimal solution requires a negative fee, i.e. the leader

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\(^8\) The output quantity \(q_{1R}\) is independent of \(r\) as a consequence of the specification of a linear demand function.
makes a positive side payment to the follower such that it abstains from producing and its payoff is not lower than what it would be in the (non-licensing) set up.

Under the (reasonable) restriction that side payments cannot be made (as, for example, in M.Katz and C.Shapiro, 1985 and in D.Sen and Y.Tauman, 2002), i.e. because (explicit) collusive behaviour is forbidden, the first best cannot obtain and the leader’s problem is:

$$\max \Pi_{1RT} = \Pi_{1R} + r q_{2R} + F$$

s.t. \( \Pi_{2R} - F \geq \Pi_2 \), and \( r, F \geq 0 \).

At the optimum, the non-negative constraint on \( F \) and the follower’s participation constraint are both binding. The solution to the leader’s optimization problem is then a pure royalty licensing contract (\( F = 0, r = r^* \)), where the optimal royalty rate \( r^* \) is such that the follower’s participation constraint holds at equality, that is \( \Pi_{2R} = \Pi_2 \).

Specifically, if the differences in costs are non drastic, then \( r^* = 1.5 \varepsilon \) and if the differences in costs are drastic, i.e. \( \Pi_2 = 0 \), then \( r^* = (1 - c + \varepsilon )/2 \).

Note that the optimal royalty rate exceeds the difference in costs \( (c_2 - c_1) \), which also implies that output (price) is lower (higher) than the one it (the leader) would obtain with no licensing. The key is as follows. The non-negative constraint on fees, \( F \geq 0 \), prevents complete collusion, i.e. optimum output is higher than the monopoly one and consequently the leader’s profits fall below the first best level. However, the royalty allows for partial collusion: optimum output is lower than that the leader would obtain with no licensing (i.e. the output that it would obtain if follower’s unit cost were \( c_2 \)). The leader does not increase its output and in order to restrain the follower’s output, the royalty is greater than the difference in costs, and the output price rises after licensing. This result does not hold in the Cournot case.

Such a neat result seems intriguing. Sequencing of moves cum royalty acts as an implicit collusive device. The sequencing of moves (which is embedded in the

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9 This follows because the derivative of the leader’s objective function with respect to \( r \) evaluated at \( r^* \) is strictly positive, i.e. at the optimum the follower’s participation constraint is binding.

9 In fact, the Cournot case \( q_{1R} \) equals \((1-c+ \varepsilon + r)/3\).

10 This result complements R.Fauì-Oller and J.Sandonis (2002) who show that the royalty may act as a collusive device when there is product differentiation and Bertrand competition.
Stackelberg-leader game) helps coordination, the appropriate royalty exploits this coordination and implements implicit (though partial) collusion. If the differences in costs are non drastic, substituting \( r = 1.5 \varepsilon \) in \((q_{1R}, q_{2R}, p_R, \Pi_{1RT}, \Pi_{2R})\), we obtain:

\[
\begin{align*}
q_{1R} &= (1 - c + \varepsilon)/2 \\
q_{2R} &= (1 - c - 2\varepsilon)/4 \\
p_R &= (1+3c)/4 \\
\Pi_{1RT} &= [(1-c+\varepsilon)^2/8] + 3\varepsilon (2- 2c - \varepsilon)/8 = \Pi_{1F} + 2F \\
\Pi_{2R} &= [(1 -c - 2\varepsilon)/4]^2
\end{align*}
\]

and the output price rises after licensing.

The leader’s total income (profits plus royalties) is always higher than the profit in the no-licensing case with non drastic differences in costs.

With drastic differences in costs, substituting \( r = [(1 - c + \varepsilon)/2] \) in \((q_{1R}, q_{2R}, p_R, \Pi_{1R}, \Pi_{2R})\) yields monopoly outcome and the follower does not produce (with and without licensing).

Summarizing the results we have the following propositions.

**Proposition 1a**
The optimal licensing contract includes a royalty and the leader licenses whenever the difference in costs is non drastic and becomes a monopoly with drastic differences in costs. In particular, in case of the optimal royalty rate, the leader charges exceeds \((c_2 - c_1)\) as the royalty acts as a collusive device.

A corollary follows.

**Corollary 1a.** Social welfare measured as the sum of consumer surplus and firms’ profits may shrink as a result of licensing.

In fact, social welfare before licensing agreements:

\[
[(3-3c+2\varepsilon)^2/32] + [(1-c+2\varepsilon)^2/8] +[(1-c-2\varepsilon)^2]/8
\]

may be larger than after licensing:

\[
1/2 [(3-3c)/4]^2 + [(1-c+\varepsilon)^2/8] + [3\varepsilon (2- 2c - \varepsilon)/8] + [(1 -c - 2\varepsilon)/4]^2
\]
if $\epsilon > (1 - c)/7$.

**Proposition 1b**
Under a fixed fee, the leader licenses its innovation to the follower if $[2(1-c)/9] > \epsilon$, i.e. its total profits after licensing are higher than those in the no-licensing case.

A corollary follows.

**Corollary 1b.** The optimal licensing contract is a royalty’s: the leader’s profits under a royalty exceed those attained by means of a fixed fee: $\Pi_{1RT} = \Pi_{1R} + r q_{2R} > \Pi_{1F} + F$.

The opposite is true for consumers, in particular $q_{2F} > q_{2R}$.

The economic intuition of this result is as follows. The leader enjoys a cost advantage under royalty licensing while the two firms compete on equal costs under fee licensing. Hence, the leader reaps the reward of licensing while still enjoying its cost advantage benefit under royalty licensing.

2.3 *The Nash bargaining solution*
An alternative would be to assume the Nash bargaining solution where the agreement maximizes the product of the rents. Assuming that firm 1 has decided to license its technology to firm 2 and charging a Nash bargained up-front fixed fee, we have to solve the following expression for the price of technology, $F$:

$$\text{Max}_F [\Pi_{1F} + F - \Pi_1][\Pi_{2F} - F - \Pi_2].$$

The values for $F$ correspond to the generalized bargaining process where the firms have all the bargaining power, respectively:

$$F_1 = 3\epsilon (2 - 2c - \epsilon)/16 \quad \text{and} \quad F_2 = \epsilon (2 - 2c + 3\epsilon)/8.$$ 

In fact, $F_1$ is equal to $F$, i.e. total surplus goes to the leader; and $F_2$ is equal to $(\Pi_{1F} - \Pi_1)$, i.e. total surplus goes to the follower.

There is a unique solution for $F$, the Nash bargaining solution, for which is equal to the mean between $F_1$ and $F_2$:
\[
F = \varepsilon \left[ 5(1 - c) + 1.5\varepsilon \right]/16.
\]

### 2.4 The follower is the innovator.

Now we consider the possibility of technology transfer from the follower to the leader under licensing. The follower allows the leader to use its cost-reducing innovation by means of a two-part tariff.

Along the lines of paragraph 2 we first consider fixed fee and a two-part tariff.

With non drastic differences in costs the maximum fee the follower can charge is:

\[
F = \Pi_{1F} - \Pi_{1i} = \varepsilon (1 - c)/2,
\]

where \(\Pi_{1i} = (1 - c - \varepsilon)^2/8\), i.e. \(\Pi_1 = (1 - 2c_1 + c_2)^2/8\) where \(c_1 = c\) and \(c_2 = c - \varepsilon\). And the follower’s total income (profits plus fixed fee) is larger than the profits in the no-licensing case, i.e. \(\Pi_{2F} + F > \Pi_2\), iff \([(1-c)/2] > \varepsilon\). The follower will license its new technology if \([(1-c)/2] > \varepsilon\), and will not if this does not hold.

We now examine two-part tariff licensing contracts. Solving the reaction functions for the follower and for the leader yields:

\[
\begin{align*}
q_1^{R'} &= (1 - c + \varepsilon - 2r)/2 \\
q_2^{R'} &= (1 - c + \varepsilon + 2r)/4 \\
p^{R'} &= (1 + 3c - 3\varepsilon + 2r)/4 \\
\Pi_{1R'} &= \left( (1 - c + \varepsilon - 2r)^2/8 \right) \\
\Pi_{2R'} &= \left( (1 - c + \varepsilon + 2r)^2/4 \right) + \left[ r (1 - c + \varepsilon - 2r)/2 \right].
\end{align*}
\]

The solution to the follower’s optimization problem is again a pure royalty licensing contract (\(F = 0\), \(r = r^*\)), where the optimal royalty rate \(r^*\) is such that the leader’s participation constraint holds at equality, that is \(\Pi_{1R} = \Pi_{1i}\).

In particular there are two values for \(r^*\): \(r_1 = \varepsilon\) for \([1-c > \varepsilon]\), and \(r_2 = (1-c)\) for \([1-c < \varepsilon]\).

The follower’s total income (profits plus royalties):

\[
\Pi_{2R'} = \left( (1 - c + 3\varepsilon ) /4 \right)^2 + \left[ \varepsilon (1 - c - \varepsilon)/2 \right]
\]

is always higher than the profits in the no-licensing case for \(r_1 = \varepsilon\).

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\[12\] The condition for the leader to stay active (non drastic innovation) is \(1-c > \varepsilon\).
Summarizing the results we have the following proposition.

**Proposition 1c**

The optimal licensing contract includes a royalty and the follower licenses whenever the difference in costs is non drastic under a fixed fee and under a royalty.

The follower’s profits under a royalty exceed those attained by means of a fixed fee:

\[
\left[\frac{1 - c + 3\epsilon}{4}\right]^2 + \left[\epsilon \frac{1 - c - \epsilon}{2}\right] > \left[\frac{1 - c + \epsilon}{4}\right]^2 + \epsilon \frac{1 - c}{2},
\]

for \(1-c > 0\).

3. A monopolist licensing

A monopolist owning a high cost technology finds it profitable to license the production of homogenous goods to a (national or foreign) firm able to produce them by a cost-reducing technology. This is a reminiscent of some results in the area of second-sourcing and licensing.

The economy is then rephrased in terms of a leader and a follower. Such case is dual to the one discussed in section 2.

Under a royalty the leader uses its own technology and licenses the production to the follower at a fixed royalty \(r\) per output unit and the latter agent will produce using its own most productive technology.

Both firms produce at constant unit production costs \(c_1\) and \(c_2\), where: \(0 < c_i < 1\) and \(c_1 > c_2\).

Along the lines of paragraph 2.2, outputs, price, profits and royalty are given by:

\[
q_{1R} = \frac{(1 - 2c_1 + c_2)\epsilon}{2} \\
q_{2R} = \frac{(c_1 - c_2)\epsilon}{2} \\
p_R = \frac{(1 + c_1)\epsilon}{2} \\
q_{1R} + q_{2R} = \frac{(1 - c_1)\epsilon}{2} \\
\Pi_{1RT} = \frac{[(c_1 - c_2)^2 + (1 - c_1)^2]/4}{4}
\]

---

15 It occurs when a firm with unique product but limited productive capacity allows other firms to produce its product under license. See J.Farrell and N.Gallini (1996) and A.Shepard (1987).
\[ \Pi_2R = [(c_1 - c_2) / 2]^2 \]

\[ r^* = [(1-c_2)/2] \]

And specifically the differences in costs are non drastic, i.e. \( q_{1R} > 0 \) and therefore \([(1+c_2)/2] > c_1 \). In particular, price and outputs are the monopoly's and the derivatives of leader's (follower's) profits with respect to follower's (leader's) costs are negative (positive).

However, an optimal licensing contract can be a two-part tariff, where \( F = [(c_1-c_2)/2]^2 \).

The comparison between leader’s total profits \( (\Pi_{1RT}) \) and monopoly’s profits \( (\Pi_{M1}) \), equal to \([(1-c_1)^2/4]\), shows that it is possible to prove the following Proposition.

**Proposition 2.** Under royalty licensing, a monopolist owning a high cost technology finds profitable to use its own technology and to license the production of homogenous goods to a firm able to produce them by a cost-reducing technology, if the differences in costs are not so drastic.

A corollary follows.

**Corollary 2.** An alternative is a procurement contract under which the firm supplies monopoly output using its own most productive technology and shares the profits with the monopolist.

In particular, monopolist’s profits are: \{\([(1-c_1)^2/4] + (1/2)\[(c_1 - c_2) (2 - c_1 - c_2)/4\}\}, and firm’s profits are: \{\[(1/2)\[(c_1 - c_2) (2 - c_1 - c_2)/4\]\}.

### 4. The output is not contractible: procurement versus licensing.

In the cases considered above output is contractible. However, in the procurement the firm can produce a higher quantity of goods and sell them on a separate market (for example in the clothing industry).\(^{16}\) It is an implicit (and often, unavoidable) part of the contract and this possibility can be taken into account in procurement. In a procurement contract the firm supplies the entire output: branded output (fixed in the contract) is sold

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\(^{16}\) Another example of an application is the pharmaceutical industry where branded and generic products co-exist.
on the primary market (1) (such as boutiques and clothes emporiums) and unbranded output is sold on a secondary market (2) (such as stock houses and local markets). Therefore, two different demand functions, as well as two cost functions, exist. In particular, the branded goods’ price equation is negatively related to their output and positively related to unbranded goods’ price and vice versa:

\[ p_1 = 1 + \theta p_2 - q_1 \quad ; \quad p_2 = 1 + \theta p_1 - q_2 , \]

where \( \theta \in (0, 1] \) and represents the degree of product differentiation.

The cost functions have constant marginal costs, \( c_1 \) and \( c_2 \).

In what follows, a procurement case is considered. In this instance, the firm supplying output for both goods typologies decides unbranded goods output depending on contract-defined branded output.

The profit functions are:

\[ \Pi_1 = (p_1 - c_1) q_1 , \]
\[ \Pi_2 = (p_2 - c_2) q_2 . \]

This case is solved backwards. Defining \( q_2 = f(q_1) \) in the second stage of the game and solving \( q_1 \) in the first stage, leads to a Stackelberg model which is solved as the ones discussed above.

Outputs, prices and profits are:

\[ q_1 = (2 + \theta + \theta c_2 - 2 c_1 + \theta^2 c_1)/(4 - \theta^2) , \]
\[ q_2 = (2 + \theta + \theta c_1 - 2 c_2 + \theta^2 c_2)/(4 - \theta^2) , \]
\[ p_1 = (2 + \theta + \theta c_2 + 2 c_1)/(4 - \theta^2) , \]
\[ p_2 = (2 + \theta + \theta c_1 + 2 c_2)/(4 - \theta^2) , \]
\[ \Pi_1 = [(2 + \theta + \theta c_2 - 2 c_1 + \theta^2 c_1)/(4 - \theta^2)]^2 , \]
\[ \Pi_2 = [(2 + \theta + \theta c_1 - 2 c_2 + \theta^2 c_2)/(4 - \theta^2)]^2 . \]

The procurement contract is compared to licensing where the licensor’s profit is a royalty on the branded output or a two part-tariff whereas the licensee decides an output mix comprising branded and unbranded goods.

The profit functions are:
\[ \Pi_{11} = r q_1 + (F), \]
\[ \Pi_{21} = (p_1 - c_1 - r) q_1 + (p_2 - c_2) q_2 - (F). \]

This case leads to a Stackelberg model too and is solved along above discussed lines.

Royalty, outputs, prices and profits are:

\[ r = \frac{(2 + \theta + \theta c_2 - 2c_1 + \theta^2 c_1)}{(4 - 2\theta^2)}, \]
\[ q_{11} = \frac{(2 + \theta - 2c_1 + \theta^2 c_1 + \theta c_2)}{[2(4 - \theta^2)]}, \]
\[ q_{21} = \frac{(8 + 6\theta - 3\theta^2 - 2\theta^3 - 8c_2 + 9\theta^2 c_2 + 2c_1 - 2\theta^4 c_2 - \theta^2 c_1)}{[2(2 - \theta^2)(4 - \theta^2)]}, \]
\[ p_{11} = \frac{(6 + 3\theta - 2\theta^2 + \theta^3 + 2c_1 - 2\theta^2 c_1 + 3\theta c_2 - \theta^2 c_2 + \theta^2 c_1)}{[(2 - \theta^2)(4 - \theta^2)]}, \]
\[ p_{21} = \frac{(8 + 6\theta - 3\theta^2 - 2\theta^3 + 8c_2 - 3\theta^2 c_2 + 2\theta c_1 - \theta^3 c_1)}{[2(2 - \theta^2)(4 - \theta^2)]}, \]
\[ \Pi_{11} = \left[\frac{(2 + \theta + \theta c_2 - 2c_1 + \theta^2 c_1)}{2}\right]^2 / \left[\frac{(2 - \theta^2)(4 - \theta^2)}{(4 - \theta^2)}\right] + (F), \]
\[ \Pi_{21} = \left[\frac{(2 + \theta - 2c_1 + \theta^2 c_1 + \theta c_2)}{2}\right]^2 + \left[\frac{(8 + 6\theta - 3\theta^2 - 2\theta^3 - 8c_2 + 9\theta^2 c_2 + 2c_1 - 2\theta^4 c_2 - \theta^2 c_1)}{(2 - \theta^2)(4 - \theta^2)}\right] / \left[\frac{(2 + \theta^2)}{2}(4 - \theta^2)\right] - (F). \]

The following proposition provides a summary of the results.

**Proposition 3.** When output is not contractible, it is more (less) profitable for the leader (follower) to choose a procurement contract than a royalty licensing, and vice versa, under a two part-tariff.

A corollary follows.

**Corollary 3.** Under no contractibility of the output, procurement may act as a barrier to overproduction of unbranded goods with respect to licensing.

In fact, given the demand functions, the ratio between branded and unbranded goods is lower in the licensing case than in the procurement.

This corollary gives some insights on different types of subcontracting.

**Conclusion**

The paper provides four major contributions to existing literature in a Stackelberg framework. The main contribution is that the optimal royalty exceeds the difference in costs between the leader and the follower. Sequencing of moves cum royalty acts as an implicit collusive device. The sequencing of moves which is embedded in the
Stackelberg game helps coordination, the appropriate royalty exploits this coordination and implements implicit (though partial) collusion. Social welfare measured as the sum of consumer surplus and firms’ profits may be therefore reduced as a consequence of the licensing agreement.

These results do not hold in the Cournot case.

Secondly, a monopolist finds it profitable to use its own high cost technology and to license the production of homogenous goods to a firm able to produce them by a cost-reducing technology.

Finally, the re-interpretation of procurement as a Stackelberg model and its comparison with licensing gives some insights on different types of subcontracting.

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