Cost reduction, profits and incentive to innovate: a note

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Abstract

In a Stackelberg (or Cournot) quantity competition model, this note shows that in the case of process innovation in both firms the profits of the leader or of the follower might decrease under simple and well accepted assumptions, and total profits might decrease.

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1. Introduction

In economic literature we meet several situations in which productivity growth is not Pareto improving.

In a classic paper J.N.Bhagwati (1958) shows that an increasing productivity of capital may reduce income due to the worsening of terms of trade.

In a joint linear production model N.Salvadori (1981) proves that the substitution of a less productive technique by a more productive one may decrease the rate of profit of the economy, given the wage rate.

J.Seade (1985) shows that in a Cournotian economy with identical firms and constant unit costs, a uniform increase in unit costs across firms may raise the industry’s profits provided that the demand curve is sufficiently convex\(^2\).

In this Note we consider an economy composed by two firms. In a Stackelberg (or Cournot) quantity competition model, it is possible to prove that in the case of process innovation in both firms (or in one of the two) the profits of one of the two agents might decrease under simple and well accepted assumptions.

If agents’ risk neutrality with respect to revenue and the possibility of transfer via side payments are assumed, it might be possible to innovate and transfers revenue. A coordination problem derives and the situation is Pareto improving.

\(^2\) See also N.van Long and A.Soubeyran (1997, pp.214-217) where the non-uniformity of costs between firms puts such a mechanism into operation.
2. Cost reduction and profits

Consider a Stackelberg quantity competition model with two firms.

Let’s assume a linear demand function:

\[ p = 1 - q \]

where: \( p \) = price of the homogenous good and
\( q \) = quantity of the good.

The cost functions for the leader (1) and the follower (2) are, respectively, equal to:

\[ c_1 = \lambda q_1 \]
\[ c_2 = \alpha q_2 \]

where \( 1 > \alpha > \lambda \).

Solving the reaction functions, quantities\(^4\), price and profits are given by:

\[ q_1 = \frac{(1 + \alpha - 2\lambda)}{2} \]
\[ q_2 = \frac{(1 - 3\alpha + 2\lambda)}{4} \]
\[ p = \frac{(1 + \alpha + 2\lambda)}{4} \]
\[ \Pi_1 = \frac{(1 + \alpha - 2\lambda)^2}{8} \]
\[ \Pi_2 = \left[\frac{(1 - 3\alpha + 2\lambda)}{4}\right]^2 \]

\(^3\) The Proposition below holds for quadratic cost functions as \( c_1 = (\lambda/2)q_1^2 \) and \( c_2 = (\alpha/2)q_2^2 \).

\(^4\) Quantities are positive if:
\[ 2\lambda - 1 < \alpha < (1+2\lambda)/3. \]
Now let’s consider process innovation and the new cost functions are:

\[ c_1 = \lambda_1 q_1 \]

\[ c_2 = \alpha_1 q_2. \]

where \( \lambda_1 < \lambda, \alpha_1 < \alpha, 1 > \alpha_1 > \lambda_1. \)

The new quantities, price and profits are:

\[ q_1 = \frac{(1 + \alpha_1 - 2\lambda_1)}{2} \]

\[ q_2 = \frac{(1 - 3\alpha_1 + 2\lambda_1)}{4} \]

\[ p = \frac{(1 + \alpha_1 + 2\lambda_1)}{4} \]

\[ \Pi_1 = \frac{(1 + \alpha_1 - 2\lambda_1)^2}{8} \]

\[ \Pi_2 = \left[\frac{(1 - 3\alpha_1 + 2\lambda_1)}{4}\right]^2 \]

It is possible to prove the following proposition:

**Proposition 1** - In a Stackelberg competition model with linear cost functions and process innovation in both firms, the profits of the leader or of the follower may decrease or increase following the rule given below.

Total profits might decrease.

Define:

\[ x = \frac{(\alpha - \alpha_1)}{(\lambda - \lambda_1)} \]  (**),
the range for increasing and decreasing profits are:\footnote{The Proposition holds in case of a Cournotian economy, i.e. in a symmetric game with asymmetric costs. For this economy the range for increasing profits is defined as \( \frac{1}{2} < x < 2 \).}:

\[
\begin{align*}
0 < x < \frac{2}{3} & \quad \Pi_1 > \Pi_1 \quad \& \quad \Pi_2 < \Pi_2 \\
\frac{2}{3} < x < 2 & \quad \Pi_1 > \Pi_1 \quad \& \quad \Pi_2 > \Pi_2 \\
2 < x & \quad \Pi_1 < \Pi_1 \quad \& \quad \Pi_2 > \Pi_2
\end{align*}
\]

Proof:

The profits of the leader are larger in the cost-reduction case with respect to the initial condition if:

\[
(1 + \alpha_1 - 2\lambda_1) > (1 + \alpha - 2\lambda), \text{ i.e. } 2 > (\alpha - \alpha_1) / (\lambda - \lambda_1).
\]

That is the condition for \( q_1 > q_1 \).

Viceversa the follower’s profit are larger in the cost-reduction case with respect to the initial condition if:

\[
(1 - 3\alpha_1 + 2\lambda_1) > (1 - 3\alpha + 2\lambda), \text{ i.e. } 2/3 < (\alpha - \alpha_1) / (\lambda - \lambda_1).
\]

That is the condition for \( q_2 > q_2 \).

Total profits might decrease. For example, if \( \alpha = 0.7, \alpha_1 = 0.6, \lambda = 0.5, \text{ and } \lambda_1 = 0.48 \) total profits decrease from 0.061875 to 0.0528.

The economic intuition of this result is explained by an externality effect. A lower \( \lambda \) has the effect to lower leader’costs, increase quantities and profits, and lower
follower’s quantities and profits. Viceversa for a lower $\alpha$. From the combined effect the results derive.

Condition (*) can be interpreted considering particular cases of cost functions.

For example let’s assume the same rate of decrease for both cost functions. Then if $\lambda > \alpha/2$ the profits of the leader will increase; viceversa for $\lambda < \alpha/2$ will decrease, while the profits of the follower will always increase.

Several corollaries follow.

**Corollary 1** - If the Stackelberg model is rephrased in terms of innovator-imitator framework and the marginal cost of the follower is lower than the marginal cost of the leader, i.e. $1 > \lambda > \alpha$, then the Proposition still holds.

**Corollary 2** - The framework considers innovations that are expensive and the Proposition holds for cost functions with fixed (a) and variable cost as:

\[
\begin{align*}
c_1 &= a + \lambda q_1 \\
c_2 &= a + \alpha q_2.
\end{align*}
\]

**Corollary 3** - In the case of zero marginal cost for the leader, i.e. $\alpha = 0$, and positive one for the follower, a process innovation in the follower’s cost function will always decrease the profits of the leader, while the profits of the follower will always increase.

**Corollary 4** - The matrix below summarizes the profits in case the leader and/or the follower innovate.

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\* In this case $\alpha/2 < \lambda < (1+\alpha)/2$ for quantities be positive.
Innovation for both firms is a dominant strategy given that Nash is played in the last stage of the game. In general if only the leader (or the follower) adopts the innovation, it will benefit and the follower (or the leader) will not.

Under the assumptions of risk neutrality with respect to revenue and transfer via side payments, it may be convenient for one firm to innovate and transfer side payments to the other. A coordination problem arises and Pareto improving is reached.
References


