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A Costly State Verification Approach

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MANAGERS’ COMPENSATION AND MISREPORTING
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Abstract

We look for the optimal shareholder-manager contract, able to induce the latter to exert high effort and truthfully reveal firm performance. This twofold incentive compatibility constraint calls for a non-linear increasing payment schedule (stock option) coupled with a verification (audit) contingent on the exercise of the option. In order to reduce the expected verification cost, the optimal stock option plan assigns the manager a large number of options with high strike price. An imperfection of the audit technology, due to the possibility of mistakes and/or collusion, is costless up to a threshold level. Beyond this level, the equilibrium contract either is distorted or it does not exist. The recent decline in the use of stock options could be interpreted in terms of poor performance of auditors. It is suggested that improving the design and efficiency of the audit activity (and supervision) is a better solution to the problem of misreporting than giving up stock options as a compensation tool.

Keywords: executive compensation, stock options, incentives, truthful revelation, auditing.

JEL codes: D82, G30, M40, M52

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1 Introduction

The use of stock options in compensation packages as a way of providing the firm management with the right incentives has been quite popular in the Nineties. The favor encountered by options started, however, to decrease in the past few years. The decline in the use of option grants in top executives compensation packages occurred mainly as a reaction to the recent accounting scandals, which has prompted the design of alternative stock-based compensation schemes (like, for example, restricted stock grants or indexed options).

As reported by Hall and Murphy (2003), the grant-date value of options assigned to all employees of an average S&P 500 firm, measured in million of 2002 constant US$, increased from 22 in 1992 to 238 in 2000 and subsequently decreased to 141 in 2002. Although percentages varied from year to year, the CEOs share of the total grant has fallen from about 7% in the mid-Nineties to less than 5% in 2000-2002. The decline in the use of options for top management is widely confirmed. According to a Pearl Meyer & Partners study for the New York Times, stock option grants, that accounted for 52% of the pay of the average CEO of a major company in 2002, were down to an estimated 35% in 2003. A recent Watson Wyatt report states that the value of new stock options awards granted to CEOs at large companies fell by nearly 60% between 2001 and 2003, with the average value of new stock options grants declining from US$ 10.2 million in 2001 to US$ 4.2 million in 2003. The decline in options has been softened by the sharp increase in restricted stock grants and other long-term incentive award values.

In this paper, we aim to reevaluate the role of stock options as an ingredient of top management compensation packages, able to align the interests

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1 A similar share accrued to the other top five executives, while the remaining managers and rank-and-file employees have received an increasing share of the total grant: from less than 85% in the mid-Nineties to over 90% by 2002.

2 The 2003 Mercer CEO Compensation Survey and Trends, analyzing the proxy statements of 350 of the largest US public companies, reported that the number of companies awarding stock options to CEOs decreased from 295 to 278 between 2002 and 2003. Over the same period, the number of CEOs receiving restricted stock grants increased from 104 to 138. Stock options dropped from 76% of the long-term incentive mix in 2002 to 62% in 2003, while restricted stock climbed from 12% to 20% and performance cash/shares rose from 12% to 18% over the same period.
of managers and shareholders, avoiding misreporting of the firm value at the same time.\(^3\) We show that incentive compatible contracts, inducing both incentive alignment and truthful revelation of the firm value, must entail a (strictly) convex compensation schedule; i.e. a fixed payment (wage) when the firm value is sufficiently low and a state contingent claim, increasing in the firm value, otherwise.

A simple and natural way to achieve a convex compensation scheme is to design a remuneration including a fix wage and a call option on the firm equity. As we will show, such remuneration allows on the one hand to preserve top management incentives (providing for a compensation increasing in equity value) and, on the other hand, to reduce the verification costs that must be incurred to avoid misreporting (as verification is not needed in the region where the remuneration is fixed).\(^4\) Other schemes, that might have otherwise desirable properties over options, do not have the convex shape discussed above. For instance, combining a fixed wage with (restricted) stock grants defines a compensation — linearly increasing in the firm (equity) value — which does not allow to reduce misreporting verification costs, that always have to be sustained in this case.

Using an incentive compatible compensation scheme (fixed wage plus call options), we characterize the contract offered by the firm under the assumption of perfect audit technology showing that, for any given value of the option package, the manager should receive the largest possible number of options at the highest strike price, allowing shareholders to reduce the probability of bearing the verification cost and hence the expected cost of the incentive compatible contract. Furthermore, the strike price of the option

\(^3\)We stress that our focus is on the use of options to provide performance-related pay to CEOs and top management only; i.e. those managers that can influence the balance sheet figures reported by the firm. Our framework does not say anything about the use of options in remunerating rank-and-file workers, that account for about 90% of all outstanding options in the US. For instance, Hall and Murphy (2003) suggest several reasons for the decline in the use of stock options, focussing on the limits and costs of options in delivering efficient results in terms of motivation, retention and attraction of employees.

\(^4\)Our argument holds for the truthful revelation problem posed by the design of the remuneration package. More precisely, we only focus on the possibility of fraudulent behavior induced by the design of the pay scheme, and not on the problems posed by fraudulent behavior per se, for whatever reasons it might arise.
turns out to be determined by incentive compatibility (i.e. the option value must be such that the manager is induced to exert the desired level of “effort”), while the magnitude of the fixed wage is defined by individual rationality.

We then turn to the problem of determining the contract characteristics under the more realistic assumption that auditing may be biased either because of the possibility of making mistakes, or because of that of collusion between manager and auditor. We label this situation as stochastic auditing. In this setting, a crucial role is played by the probability of collusion or verification mistakes. If it is low enough, nothing changes with respect to the situation with perfect auditing. However, above a certain threshold level the imperfection of the audit technology becomes costly, determining a distortion in the resulting incentive compatible contract or even preventing its existence. This result provides a rationale for the recently observed sharp decline in the use of stock options based on poor performance of auditors. We argue, however, that such remedy might rely on the wrong ingredient: a better audit should be the focus.

There are three main policy implications of our analysis. First, the current tendency of dismissing option plans in favor of other instruments is not necessarily well posed, as the use of stock options contributes to reduce the costs of aligning shareholders and managers interests. Second, as it is well known that audit resources are scarce, our model suggests that the audit activity should be selective, focussing especially — although obviously not exclusively — on the cases in which top executives exercise their options (thus giving weight to the threat of misreporting detection). Third, the same logic can be extended to the supervision of audit firms, suggesting that inspectors should pay special attention to the reports of the audit firm under scrutiny when the top management of the auditee clients benefits from substantial payments through stock-based compensation.

Overall, this paper provides two contributions, which are the two sides of a coin. On positive grounds, it explains why so many firms have dismissed stock option plans in recent years (and are still doing so), underlying that the exercise of options by top managers has not been properly monitored by auditors. On the normative side, it suggests that firms might be overreacting to accounting scandals. The way out from accounting frauds should not rely
in abandoning stock options — giving up their incentive alignment positive effects — but rather in improving the design and efficiency of the audit activity (together with the supervision on the audit sector).

The paper is organized as follows. Section 2 reviews theoretical and empirical contributions related to ours. Section 3 defines our setup, determines the characteristics of incentive compatible compensation schemes, and illustrates the optimal contract under the assumption of perfect auditing. Section 4 studies the features of the optimal contract under stochastic auditing, and Section 5 focusses on the policy implications of our contract design strategy. Section 6 summarizes and concludes.

2 Related literature

2.1 Executive compensation theory: effort, (mis)reporting, auditing

Only recently the literature on managers’ compensation has begun to deal with the interplay between performance-based payments and misreporting. The work closest to ours is perhaps the one by Goldman and Slezak (2003): they explicitly examine the trade-off between inducing high effort and creating incentive to misreport, finding that the optimal contract between shareholders and managers might not be completely free from cheating. Their analysis is, however, restricted to linear payment schedules — like a fixed wage plus some stocks. Llamazares (2004), in a multiperiod principal-agent model between firm owners and CEO, also finds that the optimal contract does not prevent the CEO from delaying the disclosure of firm losses.

Kadan and Yang (2004) explore the role of stock options in compensation packages, again finding a trade-off between effort and misreporting incentives. Dye (1988) calls the need for accepting some degree of accounting distortion — in order to preserve the effort incentive properties of managers’ compensation — the “internal demand for earnings management”, as distinct from an “external demand” — due to diverging interests between current and prospective shareholders. Furthermore, Stoughton and Wong (2003) attribute to stock options the undesirable feature of inducing “conservative accounting” (i.e. building up reserves on the balance sheet in
good times and drawing them down in bad times, in order to meet analysts’
expectations).

A related issue is the determination of the optimal strike price for ex-
cutives’ stock options. Hall and Murphy (2000) show that the valuation
problem must be carefully considered. There is a wedge between the cost
to a firm of a non-tradable stock option — which may be computed by us-
using the standard Black - Scholes formula — and its value (and incentive
properties) for a risk averse manager: the latter being lower than the for-
mer.\footnote{Ross (2004) also considers the valuation problem of a non-tradable option for a risk
averse manager, analyzing the implications for risk-taking: he finds that call options do
not necessarily create an incentive to take on more risk, contrary to the common wisdom.}
Taking this problem into account, they find that the effort-inducing
property of a stock option is maximized when the strike price is set close to
the grant date market price, justifying the practice of issuing at-the-money
 executive options. By extending this approach for considering effort aver-
sion as well, Palmon et al. (2004) find that in-the-money options should
be preferred — except for tax related disadvantages. Although our model
does not explicitly account for the value of the firm underlying assets on the
grant date, and hence we can not discuss whether stock options should be
issued in-the-money or at-the-money, we show that the strike price should
be set at the maximum level compatible with the incentive constraint.

As we are going to see (Section 5), our model has some implications for
the audit industry — and its supervision — as well. Among the several
contributions in the theory of auditing, the most relevant for our work is
the one by Kofman and Lawarrée (1993).\footnote{Let us mention here a few additional ones, dealing specifically with the collusion issue:
Baiman, Evans and Nagarajan (1991), Tirole (1986) and Antle (1984).} They analyze the role played by
an external auditor, who — contrary to the internal auditor — is involved
in a short-term contract with the firm; as a consequence, he is less prone to
collusion with the firm’s manager. The basic insight of their model is that
external auditors play a disciplining role: it may be worth to bear the cost of
hiring them, because their presence is a collusion-deterring threat between
internal auditors and managers. In this respect, Kofman and Lawarrée’s
contribution (as well as ours) points to the role of an audit activity, which
adds — in some contingencies — to the usual one taking place on a contin-

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\footnote{Let us mention here a few additional ones, dealing specifically with the collusion issue:
uous basis.\(^7\)

In modelling the information asymmetry between shareholders and man-
ger as far as accounting statements are concerned, we follow the “costly
state verification” approach, due to the seminal contributions by Gale and
Hellwig (1985) (GH), and Townsend (1979). These contributions show that
the standard debt contract may be rationalized as the optimal incentive
compatible solution to a financing problem, where investors cannot cost-
lessly observe the revenue of the firm they invest in.\(^8\) Besides, the “costly
state falsification” approach by Lacker and Weinberg (1989) must be men-
tioned. They show that an equity-like contract may emerge as an optimal
risk-sharing arrangement between two agents, when one of them can distort
— at a cost — the information about his own endowment. They also show
that optimal no-falsification contracts may be dominated by contracts that
do induce falsification. Similarly, Maggi and Rodriguez-Clare (1995) show
that costly falsification may emerge, in equilibrium, as a mean to implement
self-selection.

2.2 Empirical evidence

There is a large body of empirical literature pointing to the incentive dis-
tortions due to stock-based management compensation. We can broadly
classify such contributions into two categories, depending on the degree of
noise present in the accounting information: say a “soft” kind of misreport-
ing and a “hard” one. The former takes the form of earnings management,
often exploiting discretionary accruals;\(^9\) these leave some room for “adjust-

\(^7\)They question the optimality of the usual legal requirement of external auditing be-
ing performed annually, thus pointing to a more selective application of this additional
monitoring device (as well as we do).

\(^8\)A large body of literature has build upon the costly state verification approach. See,
for example: Innes (1990) and Dionne and Viala (1994) for an analysis of the lender-
borrower optimal contract with moral hazard; and Chang (1990) for an extension of the
GH approach to a dynamic setting. Krasa and Villamil (2000) deal with the renegotiation
issue; Mookherjee and Png (1989) introduce stochastic auditing; Diamond (1984) takes the
ex-post informational asymmetry as the basis for his well known intermediation theory;
Lacker (1998) and Bester (1985, 1994) provide a role for collateral. Literature reviews are
provided by Attar and Campioni (2003), and Freixas and Rochet (1997).

\(^9\)Accruals may be defined as the difference between the reported income and that
resulting from the firm cash flow. Take, for example, provisions for doubtful loans: an
“ing” the reported profits in order to meet a previously announced target or some analysts’ forecasts, or simply to smooth the pattern of firm performance over time. The latter takes the form of a deliberate fraud, leading to a large misrepresentation of the firm business, and it is at the origin of the more spectacular Enron-type scandals taking place in recent years. The bulk of this (cross-section) evidence is based on US data for the Nineties.\(^\text{10}\)

A first stream of literature (see, for example, Gao and Shriives, 2001; Ke, 2004; Cheng and Warfield, 2004) typically finds that managers, whose compensation has a larger share linked to their own firm’s stock price (like stock options), engage more than others in earnings management. They are also more active in trading stocks — or options on stocks — around earnings announcements, thus exploiting their informational advantage over the market.

A second group of papers looks at the evidence provided by legal procedures, initiated by the SEC (see Erickson et al., 2004; and Johnson, Ryan and Tian, 2003) or by private class actions (Peng and Roell, 2004). The emerging evidence is that stock-based managers’ compensations lead to a higher probability of a firm being classified as a “fraud firm” (meaning that a legal procedure for false or misleading statements has been initiated against such firm). Misreporting is often due to the desire of hiding a declining performance (relative to competitors).

The evidence provided by these works seems to be at odds with the traditional principal-agent view, where a performance-based compensation is seen as a beneficial tool, able to align the incentives of managers (agent) to the interests of shareholders (principal). Indeed, the evidence in favor of this incentive-alignement hypothesis dates back to the Eighties (see Brickley et al., 1985; and Lewellen et al., 1985).\(^\text{11}\)

\(^\text{10}\)The main source of data is the ExecuComp data-base provided by Standard and Poor’s, reporting compensation contracts for top managers of a huge number of US firms, starting in 1992.

\(^\text{11}\)On the other hand, Jensen and Murphy (1990) point to a small size of top managers’ performance pay, implicating that the incentive effect should be empirically irrelevant. However, as noted in the Introduction, the size of stock-based compensation has raised substantially during the Nineties.
Taken together, the principal-agent tradition and the more recent evidence on misreporting imply a clear trade-off: on the one hand, performance-based compensation packages induce managers to act in the interest of shareholders (maximize firm value); on the other hand, they introduce an incentive to manipulate accounting statements, used for measuring performance.

Strictly linked to this area of research are those papers focusing on the role of auditors. Empirical evidence suggests that the reputation of (external) auditors is an important element of corporate governance, able to affect the cost of debt (Pittman and Fortin (2004)) and the value of equity (Rauterkus and Song, 2003; Ali and Hwang, 2000). The crucial issue here is the lack of independence of audit firms from their clients, emerging from accounting scandals (the Enron-Andersen case has become the classic example, but is not the only one). This, in turn, is due to several factors: (i) the customer relationship between auditor-auditee; (ii) the real power of hiring the auditor is de facto retained by top managers, who choose their monitors; (iii) a relevant share of auditors’ compensation comes from non-audit services, introducing a clear conflict of interest (see Bajaj et al., 2003). These issues have originated a debate on how to improve the current regulation, which seems to be unsatisfactory still after the approval of the Sarbanes-Oxley Act in 2002 (see Kaplan, 2004; Rashad Abdel-khalik, 2002).

3 The basic model

Our model addresses the following issue. Shareholders have to design a compensation package, providing managers with the incentives to (i) maximize the value of equity, and (ii) correctly report the value of the firm. Such two objectives are conflicting in nature. On the one hand, the first goal requires managers’ compensation being an increasing function of the firm market value. On the other hand, an increasing payment schedule creates an incentive to overstate the value of the firm, so that a flat payment should be preferred in order to avoid accounting distortions.

3.1 Assumptions

We focus on a one-period model, where an ongoing firm is funded through equity and debt. The asset value of the firm is a random variable $\tilde{V} \in$
The face value of debt (interest payments included) is $D$, with $0 \leq V_{\text{min}} < D < V_{\text{max}}$. The controlling shareholder ($S$) — for simplicity holding the entire firm equity — delegates a manager ($M$) to run the firm. Both of them are risk-neutral. The assumption of risk neutrality for the manager is not a realistic one in many cases. In this paper, however, our focus is on the trade-off between effort (incentives) and truthful revelation by $M$, disposing of the additional risk-sharing issue (well known in the principal-agent literature) that would emerge with risk-averse managers.\(^\text{12}\)

$S$ is interested in maximizing the expected end-of-period value of equity (net of the cost of compensating $M$ and of the “verification cost” described below). In addition, he wants to avoid accounting distortions: a false balance sheet statement would trigger a loss of reputation, damaging the long run value of the firm (for instance because of a more difficult access to financial markets for funding).

$M$ chooses between two levels of effort: $e \in \{e_h, e_l\}$. High effort is more costly (disutility is directly denoted by $e_h > e_l$) but more productive, as the probability distribution of $\tilde{V}$, conditional on $e_h$, exhibits first-order stochastic dominance, defined as $F(\tilde{V} | e_h) \leq F(\tilde{V} | e_l)$ for all $\tilde{V} \in (V_{\text{min}}, V_{\text{max}})$ and $F(\tilde{V} | e_h) < F(\tilde{V} | e_l)$ for some $\tilde{V}_0$. He wants to maximize the expected value of his wage $w$ (net of the disutility of effort). His reservation level of utility is $\bar{u} \geq 0$.

The information asymmetry between the controlling shareholder and the manager is twofold. First, $S$ cannot observe the effort exerted by $M$. Second, at the end of the period $M$ (costlessly) observes the true realization of the firm asset value $V$. To the contrary, $S$ may observe $V$ only if he is willing to pay a verification cost $c$, which for simplicity is assumed to be a fixed amount.

The timing of the model is as follows. At the beginning of the period, $S$ and $M$ agree on a contract, specifying the wage ($w$) that $S$ is going to pay to $M$ at the end of the period. Then, $M$ chooses a level of effort. At

\(^{12}\)As it will become clear, assuming risk neutrality allows us to characterize a class of first best contracts without agency costs, a result that would not have been possible to achieve by introducing risk aversion for the manager. Furthermore, the assumption of risk neutrality is often adopted in the literature (see, for instance, Gale and Hellwig, 1985, and Kofman and Lawarree, 1993) as it allows to greatly simplify several analytical issues.
the end of the period, $M$ observes a realization $V$ of the firm asset value and issues a report (say a balance sheet statement) $\hat{V}$, which is publicly observed. Subsequently, $S$ decides whether to bear the verification cost $c$, in order to directly observe $V$.

Table 1 shows the market value of the firm ($V_1$), equity ($E_1$) and debt ($D_1$) at the end of the period ($t = 1$). It relies on the implicit assumption that, in case of insolvency (third line), creditors bear the verification cost, as implied by a standard debt contract (SDC). This point has already been fully developed by the literature on SDC (see Section 2.1). Therefore, we focus on the “solvency region”, where both $V$ and $\hat{V}$ are (weakly) larger than $D$: here the market value of the firm — and of its equity — depend on the choice made by $S$, as to whether implement a verification or not.

### Table 1 - End-of-period market values

<table>
<thead>
<tr>
<th></th>
<th>$V_1$</th>
<th>$E_1$</th>
<th>$D_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No-verification</td>
<td>$\hat{V} \geq D$ and $V \geq D$</td>
<td>$\hat{V}$</td>
<td>$V - D$</td>
</tr>
<tr>
<td>Verification</td>
<td>$\hat{V} \geq D$ and $V \geq D$</td>
<td>$V$</td>
<td>$V - D$</td>
</tr>
<tr>
<td>Insolvency</td>
<td>Either $\hat{V} &lt; D$ or $V &lt; D$ (or both)</td>
<td>$V$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

#### 3.2 Incentive compatible contracts

We solve the contract design problem between $S$ and $M$ in two steps. First, we define the class of incentive compatible contracts. Then, within such a class, we determine the optimal contract.

A payment schedule $w$ is a function of $V$ or $\hat{V}$, depending on whether $S$ wants to verify the report made by $M$. More formally, we define $VER$ as the values of $\hat{V}$ for which $S$ implements a verification. Then:

$$w = w(\hat{V}) \text{ if } \hat{V} \notin VER$$
$$w = w(V) \text{ if } \hat{V} \in VER$$

The payment schedule we are looking for has to satisfy two incentive compatibility constraints, in the form of a truthful revelation constraint — $\hat{V} = V$ — and in that of an effort constraint — $e = e_h$.

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13Here we are implicitly assuming that, in equilibrium, the level of effort more profitable for $S$ is indeed $e_h$. We will formally state this assumption ex post after having computed the equilibrium.
Note that, on the one hand, outside region VER there is no auditing, and therefore no detection and punishment. On the other hand, within region VER, cheating is impossible by definition. Hence, in this framework there is no need to explicitly introduce a penalty for misreporting.  

The following proposition establishes the conditions under which the truthful revelation constraint is satisfied.

**Proposition 1** \( \hat{V} = V \) if and only if the following conditions hold:

\begin{align}
  w(\hat{V}) &= \bar{w} > 0 \quad \text{constant, if } \hat{V} \notin \text{VER} \quad (1) \\
  w(V) &\geq \bar{w} \quad \text{if } \hat{V} \in \text{VER} \quad (2)
\end{align}

**Proof.** Suppose that (1) does not hold. Consider two points \( \hat{V}_0 \) and \( \hat{V}_1 \), both outside the verification region, such that \( w(\hat{V}_0) < w(\hat{V}_1) \). If \( M \) observes \( V_0 \), he reports \( \hat{V}_1 \): truthful revelation is violated. To the contrary, if \( w(\hat{V}_0) = w(\hat{V}_1) \), he has no incentive to misreport. Then, suppose that (2) does not hold. Consider a point \( V' \in \text{VER} \), such that \( w(V') < \bar{w} \). If \( M \) observes \( V' \), he does not correctly report such a value; he instead reports a \( \hat{V} \notin \text{VER} \) in order to get \( \bar{w} \). To the contrary, if \( w(V') \geq \bar{w} \) he correctly reports \( \hat{V} = V' \).

Now we come to the effort constraint. As it is typically the case with hidden effort problems, Proposition 2 shows that the payment schedule is an increasing function of the observable variable (here \( V \) or \( \hat{V} \), depending on whether we are inside or outside the verification region).

**Proposition 2** \( e = e_h \) only if \( w(\hat{V}) \) is an increasing function.

The proof of Proposition 2 is based on the following two Lemmas.  

**Lemma 1** Let \( \Delta Ew(\hat{V}) = \int_{V_{\min}}^{V_{\max}} w(\hat{V})dF(\hat{V} | e_h) - \int_{V_{\min}}^{V_{\max}} w(\hat{V})dF(\hat{V} | e_l) \). It is \( \Delta Ew(\hat{V}) = \int_{V_{\min}}^{V_{\max}} \left[ F(\hat{V} | e_l) - F(\hat{V} | e_h) \right] dw(\hat{V}) \).

---

\(^{14}\)This is going to change with stochastic auditing (see Section 4).

\(^{15}\)The Lemmas follow the logic in Hanoch and Levy (1969).
Proof. Integrating by parts $\Delta E w(\tilde{V})$ yields:

$$
\Delta E w(\tilde{V}) =
\begin{align*}
&w(\tilde{V}) F\left(\tilde{V} \mid e_h\right)\bigg|_{V_{\min}}^{V_{\max}} - \int_{V_{\min}}^{V_{\max}} F\left(\tilde{V} \mid e_h\right) \, dw(\tilde{V}) - w(\tilde{V}) F\left(\tilde{V} \mid e_l\right) \bigg|_{V_{\min}}^{V_{\max}} + \\
&\int_{V_{\min}}^{V_{\max}} F\left(\tilde{V} \mid e_l\right) \, dw(\tilde{V}) =
\end{align*}
$$

$$
= w(\tilde{V}) \left[ F\left(\tilde{V} \mid e_h\right) - F\left(\tilde{V} \mid e_l\right) \right] \bigg|_{V_{\min}}^{V_{\max}} + \int_{V_{\min}}^{V_{\max}} \left[ F\left(\tilde{V} \mid e_l\right) - F\left(\tilde{V} \mid e_h\right) \right] \, dw(\tilde{V}).
$$

It is immediate to see that the first term in the last line vanishes. □

Lemma 2 $\Delta E w(\tilde{V}) \leq 0$ if $F(\tilde{V} | e_h) \leq F(\tilde{V} | e_l)$ for all $\tilde{V} \in (V_{\min}, V_{\max})$ and $F(\tilde{V} | e_h) < F(\tilde{V} | e_l)$ for some $\tilde{V}_0$.

Proof. Using Lemma 1:

$$
\begin{align*}
F\left(\tilde{V} \mid e_l\right) - F\left(\tilde{V} \mid e_h\right) \geq 0 \Rightarrow \\
\Delta E w(\tilde{V}) &= \int_{V_{\min}}^{V_{\max}} \left[ F\left(\tilde{V} \mid e_l\right) - F\left(\tilde{V} \mid e_h\right) \right] \, dw(\tilde{V}) \leq 0
\end{align*}
$$

when $w(\tilde{V})$ is non-increasing. □

By Lemma 2 it follows immediately that $\Delta E w(\tilde{V}) - (e_h - e_l) < 0$ if $w(\tilde{V})$ is non-increasing: in this case, $M$ would choose $e_l$. Thus, $w(\tilde{V})$ increasing is a necessary condition for $M$ choosing $e_h$, which proves Proposition 2. □

Using Propositions 1 and 2, and denoting with $\tilde{V}$ a threshold level of the firm value partitioning the support of $V$ into two regions (a verification — VER — and a non-verification region), we can show:

Proposition 3 $\hat{V} = V$ and $e = e_h$ only if $w(\hat{V}) = \bar{w}$ for $\hat{V} \leq \bar{V}$, and $w(V) > \bar{w}$, $w'(V) > 0$ for $\hat{V} > \bar{V}$.

Proof. Case 1: $w = w(\hat{V})$ if $\hat{V} \leq \bar{V}$ and $w = w(V)$ if $\hat{V} > \bar{V}$. By Proposition 1 it is $w(\hat{V}) = \bar{w}$ and $w(V) \geq \bar{w}$. By Proposition 2 $w(V)$ is increasing. Thus $w(\hat{V}) > \bar{w}$, $w'(V) > 0$ ∀$V > \bar{V}$. 12
Case 2: \( w = w(\hat{V}) \) if \( \hat{V} \geq \bar{V} \) and \( w = w(V) \) if \( \hat{V} < \bar{V} \). From Proposition 1 it must be \( w(V) \geq w(\hat{V}) = \bar{w} \), implying that \( w(.) \) is decreasing in \( V \) which, by Proposition 2, violates the effort constraint \( e = e_h \). ■

The intuition behind Proposition 3 is that the verification region must coincide with high values of the firm. If the opposite happens, then Condition (2) would call for lower firm values being coupled with a higher manager’s compensation, implying a violation of the effort constraint.

Our result is complementary to that of GH, who provide a rationale for the verification carried out by debtholders in the case of insolvency (lower tail of the firm value distribution). We instead call for a verification by shareholders contingent on high realizations of the firm value (upper tail of the distribution).

Without loss of generality, we let

\[
\begin{align*}
w(\hat{V}) &= \bar{w} \text{ constant, if } \hat{V} \leq \bar{V} \\
w(V) &= \bar{w} + g(V) \text{ with } g(V) > 0 \text{ and } g'(V) > 0, \text{ if } \hat{V} > \bar{V}
\end{align*}
\]

meaning that the payment schedule has a shape as the one shown in Figure 1: a constant payment \( \bar{w} \) plus an increasing function of \( V \), for values beyond the threshold level \( \bar{V} > 0 \). This is the value delimiting the verification region: \( S \) verifies if and only if \( V > \bar{V} \).
A simple way to implement the above wage schedule is to reward the manager with a fixed payment ($\bar{w}$) together with a call option on the firm equity.\(^{16}\) For simplicity, assume that $M$ is given a European call on the firm equity, expiring at the end of the period. Define the strike price $\bar{E}$ as referred to the whole equity base of the firm (instead of a single share, as it is usually the case) and $\alpha \geq 0$ as the share of equity on which the option is written. Thus the option gives $M$ the right to buy — at the end of the period — a share $\alpha$ of the firm equity at the price $\alpha \bar{E}$. We assume that $S$ retains the fraction $(1 - \alpha)$ of the firm equity. The payment package has the following end-of-period value:

$$\bar{w} + \alpha \cdot \max \left[ E_1 - \bar{E}, 0 \right].$$ \hspace{1cm} (4)

The verification region is delimited by $\bar{V} = \bar{E} + D$: the firm value for which the option is at the money. For all realizations $V > \bar{V}$, the option is exercised and $S$ verifies the manager’s report on the firm asset value.

We assume that $\alpha$ is bounded above by an exogenous limit $\bar{\alpha}$, depending on the extent to which $S$ is willing to suffer a dilution of his equity stake in the firm.\(^{17}\)

### 3.3 Optimal contract

So far, we have delimited the class of incentive compatible contracts. In particular, we have identified a contract type based on a fixed payment $\bar{w}$ plus a call option on a share $\alpha$ of the firm equity with strike price $\bar{E}$. If the option is in the money, this triggers a verification of the balance sheet statement issued by the manager.

\(^{16}\)It is worth to note that we focus on stock options as they seem to be the most natural way to achieve a strictly convex compensation schedule, but they are by no means the only one. It is in fact possible to conceive other stock-based remuneration packages providing a fix wage up to a certain threshold level of the firm value and an increasing pay thereafter. Think, for example, to performance related pay based on targets defined upon cash flows, net income growth, return on capital employed or on equity, increases in market share and stock grants conditional on a target stock price.

\(^{17}\)We are also implicitly assuming that renegotiation is not allowed. This can be a restrictive assumption, as stock options are often repriced before maturity. However, allowing for repricing — making an option more likely to be exercised — does not alter the nature of our problem and our results, at least qualitatively.
We now need to select, within this class, the optimal contract. This is defined by the values of $\bar{w}$, $\bar{E}$ and $\alpha$ such that the total cost for $S$ to induce $M$ to exert the high level of effort is minimized. The optimal contract is more conveniently described as a triple $(\bar{w}^*, \bar{V}^*, \alpha^*)$; the optimal strike price being given by $\bar{E}^* = \bar{V}^* - D$\textsuperscript{18}. In order to guarantee that stock options can be used to provide incentive alignment, we assume that parameter values are such that $\bar{E}^* \geq 0$, i.e. $\bar{V}^* \geq D$. As we will see after having characterized the optimal contract, this assumption has a natural interpretation. Note since now, however, that if this technical condition were violated, then options would not exist as an effort inducing compensation tool, which is of course counterfactual.

At the beginning of the period ($t = 0$), $S$ solves the following Problem 1, in order to determine the optimal contract:

**Problem 1**

\[
\min_{\bar{w}, \bar{V} \geq D, \alpha \leq \bar{\alpha}} \bar{w} + \int_{\bar{V}}^{V_{\text{max}}} \alpha (\bar{V} - \tilde{V}) f(\tilde{V} | e_h) d\tilde{V} + \Pr(\bar{V} \geq e_h) \cdot c \quad \text{(OF)}
\]

\[s.t.:
\]

\[
\int_{\bar{V}}^{V_{\text{max}}} \alpha (\bar{V} - \tilde{V}) f(\tilde{V} | e_h) d\tilde{V} - \int_{\bar{V}}^{V_{\text{max}}} \alpha (\bar{V} - \tilde{V}) f(\tilde{V} | e_l) d\tilde{V} \geq e_h - e_l \quad \text{(IC)}
\]

\[
\bar{w} + \int_{\bar{V}}^{V_{\text{max}}} \alpha (\bar{V} - \tilde{V}) f(\tilde{V} | e_h) d\tilde{V} \geq \bar{u} + e_h \quad \text{(IR)}
\]

where the total cost for $S$ of implementing $e_h$ (fixed payment plus option value plus expected verification cost) is minimized, taking into account two constraints. The first one is an incentive compatibility constraint: $M$ has to be induced to exert a high level of effort.\textsuperscript{19} The second one is an individual rationality constraint, requiring that $M$ receives at least his reservation level of utility.

\textsuperscript{18}This technical choice, due to the fact that the density function $f(\cdot)$ is defined over $\bar{V}$, avoids a change of variable in the following analysis.

\textsuperscript{19}Here we take care only of the effort constraint; the truthful revelation constraint is satisfied thanks to the shape of the payment schedule. Such a shape is necessary to get high effort (see Proposition 2), while it is necessary and sufficient to get truthful revelation (see Proposition 1). $f(\tilde{V} \mid \cdot)$ denotes the p.d.f. of $\tilde{V}$, conditional on each effort level.
Solving Problem 1 under the assumptions made above, we can state the following:

**Proposition 4** In equilibrium both (IC) and (IR) hold with equality.

There exists an optimal contract \([\alpha^*, \tilde{V}^*(\alpha^*), \bar{w}^*(\alpha^*)]\) implicitly defined by

\[
\alpha^* = \bar{\alpha} \quad (5)
\]

\[
\tilde{V}^* := \tilde{V}^*(\alpha^*) = \frac{\max_{\tilde{V}} \tilde{V} \left( f(\tilde{V} | e_h) - f(\tilde{V} | e_l) \right) d\tilde{V} - \frac{\bar{w}^* - \bar{u}^*}{\alpha^*}}{\max_{\tilde{V}} \left( f(\tilde{V} | e_h) - f(\tilde{V} | e_l) \right) d\tilde{V}} \quad (6)
\]

and

\[
\bar{w}^* := \bar{w}^*(\alpha^*) = \bar{u} + e_h - \int_{\tilde{V}^*}^{V_{\text{max}}} \alpha^* (\tilde{V} - \tilde{V}^*) f(\tilde{V} | e_h) d\tilde{V}. \quad (7)
\]

**Proof.** In equilibrium, Constraint (IR) is binding. Suppose not: \(\exists \tilde{w}' < \bar{w}\) such that (IR) is satisfied and the objective function (OF) assumes a lower value. Therefore, \(\tilde{w}'\) can not be an equilibrium value.

Restate Problem 1 by substituting (IR) into (OF). Hence, the problem becomes \(\min_{\tilde{V} \geq D, \alpha \leq \bar{\alpha}} \Pr(\tilde{V} \geq \tilde{V} | e_h) \cdot c\) subject to (IC), or equivalently \(\max_{\tilde{V} \geq D, \alpha \leq \bar{\alpha}} \tilde{V} \) subject to (IC). Denote by \(L(\tilde{V}, \alpha)\) the LHS of (IC). \(L(\tilde{V}, \alpha)\) is decreasing in \(\tilde{V}\), for

\[
\partial L(\tilde{V}, \alpha) / \partial \tilde{V} = -\alpha \left( \int_{\tilde{V}}^{V_{\text{max}}} f(\tilde{V} | e_h) d\tilde{V} - \int_{V_{\text{max}}}^{\tilde{V}} f(\tilde{V} | e_l) d\tilde{V} \right) = \quad (8)
\]

\[
\alpha \left[ F(\tilde{V} | e_h) - F(\tilde{V} | e_l) \right] < 0
\]

by stochastic dominance. In equilibrium, Constraint (IC) is binding. Using the same logic as above, suppose not: \(\exists \tilde{V}' > \tilde{V}\) such that (IC) is satisfied and the shareholder is better off. Note also that it must be \(\tilde{V}^* < V_{\text{max}}\), for (IC) to be met.; this is without loss of generality as \(V_{\text{max}}\) can be set exogenously arbitrarily large.
\( V^* (\alpha) \) is increasing in \( \alpha \): by applying the implicit function theorem to Constraint (IC) with equality, it is

\[
\frac{\partial V^* (\alpha)}{\partial \alpha} = - \frac{\int_{V} [f(\tilde{V} | e_h) - f(\tilde{V} | e_l)](\tilde{V} - V) d\tilde{V}}{\alpha[F(V^* (\alpha) | e_h) - F(V^* (\alpha) | e_l)]} > 0, \tag{9}
\]
as the numerator is positive by the same arguments as in the proof of Lemmas 1 and 2 and the denominator is negative by stochastic dominance. Thus, it is \( \alpha^* = \bar{\alpha} \).

Finally, by substituting \( \tilde{\alpha} \) and \( \tilde{V}^* (\tilde{\alpha}) \) into (IR) with equality, we obtain (7), which is positive provided \( \tilde{u} \) is sufficiently large.

Note that, substituting for \( \tilde{V}^* \) from Equation (6) and rearranging, the assumption that \( \tilde{V}^* \geq D \) can be stated as

\[
\bar{\alpha} \int_{\tilde{V}^*}^{V_{\infty}} \left( \tilde{V} - D \right) \left( f(\tilde{V} | e_h) - f(\tilde{V} | e_l) \right) d\tilde{V} \geq e_h - e_l,
\]
meaning that exerting high effort must be worthwhile, generating a sufficient increase in the value of equity assigned to the manager through the stock option plan.

Proposition 4 states that the optimal contract has the following features. First, the equilibrium level of \( \alpha \) is the upper bound \( \bar{\alpha} \): so \( \alpha^* = \bar{\alpha} \). Intuitively, by giving the manager a larger number of options, the shareholder is able to increase the strike price, for any given value of the option package. By doing so, the shareholder reduces the probability of bearing the verification cost, hence the expected cost of implementing an incentive compatible contract.

Second, the optimal strike price \( \bar{E}^* = \tilde{V}^* - D \) is determined by the incentive compatibility constraint (holding with equality in equilibrium); then the optimal fixed payment \( \bar{w}^* \) is determined by the individual rationality constraint (with equality). The intuition is the following. The wage schedule is defined by two components: a fixed payment plus a state contingent claim (option). The latter “takes care” of the incentive problem, while the former ensures that \( M \) is willing to sign the contract. In other words, the option value must be such that \( M \) is induced to exert a high level of effort,
so that incentive compatibility puts a constraint on the strike price. On the other hand, the fixed payment $\bar{w}$ “takes care” of the individual rationality constraint.\footnote{Our result of setting options strike price at the maximum level compatible with the effort constraint is not directly comparable with that, advocated by other contributions (e.g. Hall and Murphy, 2000), of issuing at-the-money options, as we do not explicitly consider the value of the underlying assets at grant date. The discrepancy between such result and ours follows from the structure of our twofold incentive compatibility problem, dealing both with an effort and a truthful revelation constraint, while Hall and Murphy focus on the design of a contract maximizing the effort inducing properties of stock options.}

In order to guarantee that it is in $S$ interest to have $M$ exerting the high level of effort, the following condition must be met:

$$
V_{\text{max}} \int_{D} (\tilde{V} - D) \left[ f(\tilde{V} | e_{h}) - f(\tilde{V} | e_{l}) \right] d\tilde{V} \geq e_{h} - e_{l} + \Pr(\tilde{V} \geq \tilde{V}^{*} | e_{h}) \cdot c, \quad (10)
$$

which ensures that the gain for $S$ from inducing high effort more than compensates its cost. To see why, note that $S$ has an interest in inducing high effort from $M$ if

$$
V_{\text{max}} \int_{D} (\tilde{V} - D) f(\tilde{V} | e_{h}) d\tilde{V} - \bar{w}^{*} - \bar{\alpha} \int_{\tilde{V}^{*}} (\tilde{V} - \tilde{V}^{*}) f(\tilde{V} | e_{h}) d\tilde{V} - \Pr(\tilde{V} \geq \tilde{V}^{*} | e_{h}) \cdot c \geq 0,
$$

where $\bar{w} (e_{l}) = \bar{u} + e_{l}$ is the constant payment to $M$ when the signed contract requires low effort. By substituting for $\bar{w}^{*}$ from (7), plain algebra yields Inequality (10).

Finally, one can easily check that Problem 1 with no verification costs (i.e. $c = 0$) — the benchmark case in which the principal ($S$) perfectly and costlessly observes the true firm value $V$ — is such that the incentive compatibility constraint (IC) is not binding, implying that the hidden effort problem does not generate agency costs, as expected for the agent ($M$) is risk neutral.\footnote{As (IR) is binding, Problem 1 reduces to minimize $(\bar{u} + e_{h})$ subject to (IC). As both $\bar{u}$ and $e_{h}$ are exogenously given, the result follows immediately.} In this case, there are many first best contracts since the values of $\bar{u}$ and $\bar{V}$ are undetermined, while the optimal level of $\bar{w}$ is
uniquely determined by the (binding) individual rationality constraint (IR). It remains true that the compensation package offered to the manager must be increasing in the firm value $V$, as required by Proposition 2. Therefore, with $c > 0$, the agency cost associated to truthful revelation is the expected verification cost, which is a deadweight loss.

4 Stochastic auditing

So far, we have implicitly assumed that the audit technology available to the controlling shareholder is perfect: $S$ observes the true value $V$, provided he is willing to bear the verification cost $c$. In this section, we explore the consequences of a more realistic assumption, namely that the audit technology is imperfect. There are at least two reasons why this can be the case: on the one hand, the audit process might lead to a mistake and, on the other hand, the auditor and the manager might collude. In both cases the auditor will end up misreporting the firm value, either purposively or in good faith, with a positive probability.

In the following, we model the information provided by the verification as a noisy signal: $R \in \{T, F\}$. Suppose that, when $S$ decides to carry out a verification, he delegates this task to an auditor, whose report is $R = T$, meaning that the balance sheet statement issued by the manager is truthful ($\hat{V} = V$), or $R = F$ meaning that the balance sheet statement is false ($\hat{V} \neq V$). When it is indeed $\hat{V} = V$, the auditor always reports $R = T$. However, when $\hat{V} \neq V$ he reports $R = F$ with probability $p \in (0, 1)$. In other words, with probability $(1 - p)$ either he makes a mistake — as he does not detect a false statement made by the manager — or he deliberately hides misreporting as the outcome of a collusive agreement with the manager. Formally: $\Pr(R = T|\hat{V} = V) = 1$ and $\Pr(R = F|\hat{V} \neq V) = p < 1$. We assume the detection probability $p$ to be exogenously given.

As for the consequences of the verification, if the auditor reports $R = T$, the manager is given his contractual payment. If to the contrary $R = F$, the manager does not receive such a payment. In addition, $M$ suffers a penalty — denoted by $P$ — that we assume to be increasing in the amount of misreporting, i.e.

$$P := \gamma |\hat{V} - V|,$$

(11)
where \( \gamma > 0 \) is a parameter capturing the severity of the punishment.\(^{22}\) We may think of this penalty as the loss of job and reputation by \( M \), or as legal prosecution by shareholders. The definition and size of \( P \) is assumed to be set by the law (and/or the environment), and not endogenously determined by \( S \) as a solution of his decision problem. It is important to stress that our model does not offer insights on the design of the penalty suffered by \( M \). Punishment is here modeled in a very simple way, without dealing with issues related to the enforceability (that is assumed to be always satisfied) and to the optimal design of the punishment scheme.\(^{23}\)

We solve the manager’s problem in two steps. First, we determine the optimal amount of misreporting, given that \( M \) decides to cheat. Second, we determine whether \( M \) actually decides to misreport or not.

**Step 1.** Taking \( P \) as given, the (risk-neutral) manager who has decided to misreport solves the problem

\[
\max_{\hat{V}} - p \gamma (\hat{V} - V) + (1 - p) \left( \alpha \left(\hat{V} - \overline{V}\right) + \overline{w}\right),
\]

i.e.

\[
\max_{\hat{V} \geq V} (\alpha (1 - p) - p \gamma) \hat{V} + p \gamma V - \alpha (1 - p) \overline{V} + (1 - p) \overline{w}. \tag{12}
\]

In writing Problem (12) we stick without loss of generality to the case in which \( \hat{V} \geq V \), as in our model a misreporting manager has never an incentive to falsely state a firm value below the true one (the only incentive to misreport is to inflate the value of the stock option plan).

\(^{22}\) Alternatively, we could consider the case in which the detection probability \( p \) (instead of the punishment \( P \)) is endogenous and of the form

\[
p = \beta \frac{\hat{V} - V}{V_{\text{max}} - V},
\]

with \( 0 \leq \beta \leq 1 \) a parameter denoting the efficiency of the audit technology and \( \hat{V} \geq V \) without loss of generality. In this case it is the probability of detection to be increasing in the magnitude of misreporting. It is easy to show that our results would remain qualitatively the same: what matters in fact is that the expected punishment is increasing in the amount of misreporting.

\(^{23}\) These are major points for the literature on crime (see, for instance, Becker, 1968; Stigler, 1970; and Andreoni, 1991), which we dispose of as they are not central to our argument. A drawback of our modeling choice, however, is that we can not derive implications on the optimal value of the severity (\( \gamma \)) of punishment.
By inspection of Problem (12), it is immediate that 
\[ \hat{V}^* = V_{\text{max}} \]
if
\[ \alpha (1 - p) - p\gamma > 0, \] i.e.
\[ p < \bar{p} := \frac{\alpha}{\alpha + \gamma}, \]
and \( \hat{V}^* = V \) otherwise.

Observe that when \( p \geq \bar{p} \), the truthful revealing constraint is automatically satisfied as truthfully reporting is optimal for \( M \), and the shareholder simply solves Problem 1 in Section 3.3. In this case, in fact, it must be that the expected value of the penalty is larger than (or equal to) the expected gain from misreporting.

Consistently with the above analysis, in the remaining of this section we focus on the case in which \( p < \bar{p} \), so that the manager has, in principle, an incentive to cheat (reporting \( \hat{V}^* = V_{\text{max}} \)).

**Step 2.** Proposition 1 is no longer sufficient for truthful revelation (while it still gives necessary conditions). From Definition (11), and given that \( \hat{V}^* = V_{\text{max}} \), it follows immediately that \( P^* = \gamma (V_{\text{max}} - V) \). The condition which guarantees a truthful statement by the manager is the following:

\[ \bar{w} + \alpha \cdot \max \{ V - \hat{V}, 0 \} \geq (1 - p) \left[ \bar{w} + \alpha (V_{\text{max}} - \hat{V}) \right] - pP^*, \]

where the LHS is the payoff \( M \) gets if he correctly reports the observed asset value \( V \), while the RHS is his expected payoff if he decides to cheat, by reporting \( V_{\text{max}} \neq V \). Depending on whether the call option in the manager’s compensation package is in-the-money, Condition (14) can be written as:

\[ \alpha (V_{\text{max}} - \hat{V}) \leq \frac{p}{1 - p} (\bar{w} + P^*) + \alpha \frac{1}{1 - p} (V - \hat{V}) \] for \( V > \hat{V} \),

\[ \alpha (V_{\text{max}} - \hat{V}) \leq \frac{p}{1 - p} (\bar{w} + P^*) \] for \( V \leq \hat{V} \).

By assuming that Condition (10) — evaluated at the new equilibrium — holds, we are now in the position to state the optimal contract design problem with stochastic auditing. At \( t = 0 \), \( S \) has to solve the following Problem 2:

**Problem 2**

\[
\min_{\bar{w}, \bar{V} \geq D, \alpha \leq \bar{\alpha}} \bar{w} + \int_{\bar{V}}^{V_{\text{max}}} \alpha (\bar{V} - \bar{V}) f(\bar{V} | e_h) d\bar{V} + \Pr(\bar{V} \geq \bar{V} | e_h) \cdot c \quad (\text{OF})
\]
where an additional constraint has been added, relative to Problem 1 in Section 3.3, namely the truthful revelation condition (TR). Note that only Condition (16), where \( P^* = \gamma (V_{\text{max}} - V) \), has been introduced here: the reason is that Condition (15) holds whenever (16) does, so only the latter actually matters. Note also that Problem 1 may be seen as a particular case of Problem 2, where \( p \to 1 \): in this case, (TR) is trivially satisfied (even if \( P = 0 \)).

By solving Problem 2, we can prove the following:

**Proposition 5** Let \( p < \bar{p} \), where \( \bar{p} \) is determined in Equation (13), and define \( \bar{p}(\alpha) \) such that (TR) is binding.

Case 1. \( \bar{p}(\bar{\alpha}) \leq p < \bar{p} \): the equilibrium (second best) contract \( \{\bar{\alpha}, \bar{V}^*(\bar{\alpha}), \bar{w}^*(\bar{\alpha})\} \) is the one derived in Proposition 4.

Case 2. \( p < \bar{p}(\bar{\alpha}) < \bar{p} \) or \( \bar{p} < \bar{p}(\bar{\alpha}) \). If there exists a value \( \alpha^{**} \), defined as the maximum value of \( \alpha < \bar{\alpha} \) such that (TR) is binding, then the solution to

\[
\begin{align*}
\int_{\bar{V}}^{V_{\text{max}}} \alpha(\bar{V} - \bar{V}) f(\bar{V} | e_h) d\bar{V} - \\
\int_{\bar{V}}^{V_{\text{max}}} \alpha(\bar{V} - \bar{V}) f(\bar{V} | e_l) d\bar{V} & \geq e_h - e_l \quad \text{(IC)} \\
\bar{w} + \int_{\bar{V}}^{V_{\text{max}}} \alpha(\bar{V} - \bar{V}) f(\bar{V} | e_h) d\bar{V} & \geq \bar{u} + e_h \quad \text{(IR)} \\
\frac{p}{1-p} (\bar{w} + P^*) - \alpha (V_{\text{max}} - \bar{V}) & \geq 0 \quad \text{(TR)}
\end{align*}
\]

\[24\text{Consistently with our assumption in Section 3.1, in stating the problem we are implicitly assuming that the controlling shareholder } S \text{ has an incentive to guarantee truthful revelation from the manager. This amounts to assume that } S \text{ suffers a cost following misreporting (in terms, for example, of the consequences of a loss of reputation) sufficiently high to provide him with the incentive to design a compensation package ensuring truthful revelation. This does not necessarily need to be the case. In principle, the firm faces a trade-off between the cost (loss of reputation) incurred in case of detected misreporting and the “cheaper” compensation package the firm can offer when not taking truthful revelation into account. Whether } S \text{ will ensure truthful revelation or not depends on the impact of the misreporting cost.}\]

\[25\text{This is the reason why we did not explicitly introduce a penalty for cheating in the basic model of Section 3.}\]
the contract design problem is a (third best) contract \{\alpha^{**}, \bar{V}^*(\alpha^{**}), \bar{w}^*(\alpha^{**})\},
where

\[
\alpha^{**} = \frac{p \bar{w}^*(\alpha^{**}) + P^*}{1 - \bar{p} (V_{\text{max}} - \bar{V}^*(\alpha^{**}))},
\]

(17)

and \(\bar{V}^*(\alpha^{**}), \bar{w}^*(\alpha^{**})\) follow from (IC) and (IR) with equality respectively. Otherwise, a contract with \(\alpha \leq \bar{\alpha}\) and satisfying both (IC) and (TR) does not exist.

**Proof.** As shown in Proposition 4 and for the same reasons, Constraints (IR) and (IC) are binding. It is thus possible to solve (IR) and (IC) for \(\bar{w}^*(\alpha)\) and \(\bar{V}^*(\alpha)\) respectively. Denote with \(H(p, \alpha)\) the LHS of Constraint (TR) after substituting for \(\bar{w}^*(\alpha)\) and \(\bar{V}^*(\alpha)\). For all \(\alpha \leq \bar{\alpha}\) there exists \(\hat{\bar{p}}(\bar{\alpha}) \in (0, 1)\) such that \(H(\hat{\bar{p}}(\bar{\alpha}), \alpha) = 0\) so that (TR) is binding. As \(\frac{\partial \bar{V}^*(\alpha)}{\partial \alpha} > 0\) (see Equation 9) and given that the objective function in Problem 2 is decreasing in \(\bar{V}\), \(S\) will select the highest \(\alpha < \bar{\alpha}\) compatible with all the problem constraints. Thus, we focus on \(\hat{\bar{p}}(\bar{\alpha})\) as a benchmark. Assume \(\hat{\bar{p}}(\bar{\alpha}) \leq p < \bar{\bar{p}}(\bar{\alpha})\) (Case 1), \(H(p, \bar{\alpha}) \geq 0\) so that (TR) is weakly satisfied: Problem 2 reduces to Problem 1 and the optimal contract, \{\(\bar{\alpha}, \bar{V}^*(\bar{\alpha}), \bar{w}^*(\bar{\alpha})\}\), is the one illustrated in Proposition 4. Consider now \(p \leq \hat{\bar{p}}(\bar{\alpha}) < \bar{\bar{p}}\) or \(\bar{\bar{p}} < \hat{\bar{p}}(\bar{\alpha})\) (Case 2), recalling that we stick to the case \(p < \bar{\bar{p}}\) without loss of generality. \(H(p, \bar{\alpha}) < 0\) and thus (TR) is violated for \(\alpha = \bar{\alpha}\). Suppose there exists a value \(\alpha^{**}\), defined as the maximum value of \(\alpha < \bar{\alpha}\) such that \(H(p, \alpha^{**}) = 0\), i.e. such that Constraint (TR) is binding. There exists a (third best) solution \{\(\alpha^{**}, \bar{V}^*(\alpha^{**}), \bar{w}^*(\alpha^{**})\}\) to the contract design problem: (17) follows immediately from (TR), \(\bar{V}^*(\alpha^{**})\) and \(\bar{w}^*(\alpha^{**})\) from (IC) and (IR), respectively. Finally, if \(\alpha^{**} < \bar{\alpha}\) does not exist, Constraint (TR) is violated for all admissible \(\alpha\) and a contract inducing high effort and truthful revelation can not exist. ■

Note that what is crucial for the argument developed in Proposition 5 is the behavior of the manager compensation package with respect to the equity share on which his performance related pay is based \(\alpha\). On the one hand, when \(\alpha\) increases, the size of the stock option package rises. On the other hand, also the option strike price \(E^* = \bar{V}^*(\alpha) - D\) increases — as \(\partial \bar{V}^*(\alpha)/\partial \alpha > 0\) — which implies that the unit value of the option plan
drops. The overall effect on the value of the option package is undetermined. Considering also that \( \tilde{w}^* \) is a function of both \( \alpha \) and \( V^*(\alpha) \), the relationship between \( \alpha \) and \( H(p, \alpha) \) is not necessarily monotonic. Therefore, it is impossible to guarantee the existence of an \( \alpha^{**} < \bar{\alpha} \) solving the contract design problem. Whether \( \alpha^{**} \) exists depends clearly on parameter values, being thus an empirical matter that can not be established in general.

Proposition 5 implies that, as long as the audit technology is not too noisy (i.e. the efficiency \( p \) of the audit technology is above a threshold level \( \tilde{p} \)), the optimal contract under stochastic auditing is the same as under perfect auditing. In particular, the optimal level of \( \alpha \) is still given by the exogenous upper bound \( \bar{\alpha} \), and the optimal payment schedule is given by Equations (6)-(7) in Proposition 4 of Section 3.3. The intuition is that, if the probability of making verification mistakes is low enough, the truthful revelation constraint is not binding; hence, Problem 2 collapses into Problem 1. In other words, an imperfection of the audit technology is costless, up to the threshold level \( \tilde{p} \).

To the contrary, beyond such level the imperfection becomes costly, and it might even prevent the existence of an incentive compatible contract. The possibility of non-existence is not surprising, for our contract design strategy relies on two ingredients: performance related pay, coupled with auditing. If the latter is too inefficient, such a recipe fails.

There are two quite different reasons for the possibility of non-existence indeed. First, a level of \( \alpha \) satisfying both the incentive compatibility and the truthful revelation constraints might not exist. In this case, shareholders face a choice between an effort inducing contract and a truthfully revealing one: this result highlights the tension between the (IC) and (TR) constraints, the former calling for a high value of the option package, the latter pointing to a flat compensation scheme. Second, such level of \( \alpha \) might exist but be greater than \( \bar{\alpha} \), hence requiring shareholders to give up a fraction of equity larger than they are willing to.

Note finally that, when existing, the optimal contract in presence of stochastic auditing defines an inferior (third best) situation with respect to the (second best) contract with perfect auditing. If the probability of detecting misreporting is low enough (\( p \leq \tilde{p} \)) the audit technology is too noisy: in this region, (TR) is binding and it requires a distortion of the
equilibrium contract, imposing to lower $\alpha$ below $\bar{\alpha}$.

Proposition 5 has two major implications. First, it confirms that options are a viable tool to align shareholders and top executives incentives, provided that the audit technology is not too inefficient; in particular, imperfect auditing does not necessarily undermine the efficacy of options. Second, it suggests an explanation for the recent sharp decrease in the use of stock options in top managers compensation: available audit mechanisms are presumably too noisy, so that contracts based on options lead to misreporting. We suggest that firms might be reacting in the wrong direction: instead of disposing of stock options altogether, they should devote more resources to improve the audit associated with the exercise of options.

5 Policy implications

Contract design between shareholders and managers faces two conflicting incentive-compatibility constraints: (i) induce high effort; (ii) induce truthful revelation of the firm performance. Our solution to the problem relies in a non-linear payoff schedule — of the stock option type — coupled with a verification, contingent on the exercise of the option. This is the way to minimize the verification cost, preserving incentive compatibility — provided the auditing technology is not too noisy.

This message has several policy implications. First, stock options should not be dismissed: they are a useful tool for aligning the incentives of managers to those of shareholders; then they should be part of the compensation package. Thus, recent corporate scandals should not lead to the conclusion that stock options are to be banned altogether.

Second, the audit activity should be selective. It is well known that a relevant problem external auditors face is the paucity of (human) resources devoted to the audit process: as such resources are costly, an auditor is typically forced to meet very strict time constraints in examining the accounts of his clients (see Kaplan, 2004). This leads to implement sampling criteria, where only a fraction of the client’s transactions are actually examined. Our model suggests instead that scarce audit resources should be focussed on those cases where the top managers of the auditee firm exercise the stock options they hold (as part of their compensation package): this would help
increasing the efficiency of the audit industry.\textsuperscript{26}

Third, \textit{the supervision on audit firms should be selective} as well. Recent corporate scandals have lead legislators to strengthen the supervisory framework of the audit industry. In USA, the Sarbanes-Oxley Act of 2002 (S-O Act) established the Public Company Accounting Oversight Board (PCAOB), which is empowered - among other things - to conduct inspections of registered accounting firms.\textsuperscript{27} In Europe, the Commission has proposed a Directive, requiring member States to implement a quality control system on audit firms.\textsuperscript{28} Of course, inspections cannot replicate the whole activity of the audit firm under scrutiny; it is necessary to select a small number of deals to be checked out.\textsuperscript{29} Our model suggests a way (not necessarily the only one) to select those cases which should prominently attract the attention of the supervisors: when the managers of the auditee client benefit from large payments in the form of (in-the-money) stock options, they have a greater incentive not only to misreport the firm value, but also to put pressure on (or collude with) the auditor in order to gain a benevolent audit report.

\textsuperscript{26}Obviously, we are not saying that \textit{only} such cases should be examined by auditors. There are indeed many other possible distortions we do not deal with that the auditor should take care of.

\textsuperscript{27}See Section 104 of the S-O Act. Annual inspections are required for firms auditing more than 100 issuers; at least triennial inspections for others. In particular, the PCAOB should “inspect and review selected audit and review engagements of the firm” and “evaluate the sufficiency of the quality control system of the firm”.

\textsuperscript{28}See the Commission’s Proposal for a Directive on statutory audit of annual accounts and consolidated accounts (COM-2004-177-Final, March 16 2004). In particular, “Member States shall ensure that all statutory auditors and audit firms are subject to a system of quality assurance...organized in a manner that is independent from the reviewed statutory auditors and audit firms and subject to public oversight” (Art.29). The quality review must be carried out at least every three years for those firms auditing “public interest entities” (including listed firms and financial intermediaries); otherwise, at least every six years.

\textsuperscript{29}Consider that 1,271 accounting firms are registered with the PCAOB (as of August 2004). In the “Reports on 2003 Limited Inspections”, the criteria used to select those audit engagements to be reviewed are not disclosed.
6 Concluding remarks

Both recent empirical evidence and theoretical contributions in corporate governance point to the existence of a trade-off between incentive alignment and misreporting: performance-related pay induces managers to act in the interest of shareholders, but it also creates an incentive to inflate accounting reports.\textsuperscript{30} In this paper, we try to answer the basic question on how should a compensation package be designed in order to get both incentive alignment and truthful reporting. Our solution proposes a strictly convex payment schedule, which can be implemented for example through a fixed wage coupled with a stock option plan. The increasing region of the contract (when the option is in-the-money) triggers a verification by shareholders. Of course, this is a second best outcome, as a verification cost must be sustained; but the contract is designed in such a way as to minimize its expected value by giving the manager the largest possible number of options at the highest strike price.

On theoretical grounds, this result complements the well known Gale and Hellwig (1985) theory of debt as a way to solve a costly state verification problem. In their seminal contribution, the optimal debt-like contract is such that the debt-holder receives a strictly concave payoff, and he implements a verification (bankruptcy procedure) in the \textit{lower} tail of the firm value distribution. To the contrary, our shareholder - manager delegation problem leads to a verification region which coincides with the \textit{upper} tail of the distribution.

In terms of policy, our model suggests that options are a useful tool for remunerating top managers, provided their exercise is coupled with a specific audit activity. This approach provides a criterion for allocating scarce audit resources: auditors should pay attention particularly (although not exclusively) to those firms whose managers have exercised their stock options. The same principle applies to the supervision over the audit industry: supervisors (e.g. the PCAOB in the U.S.) should focus on audit reports relative to those firms whose top managers have substantially benefitted from stock based compensation, also because this might increase the likelihood of collusive agreements between auditor and auditee.

\textsuperscript{30}See the literature surveyed in Section 2.
Recent empirical evidence shows that firms have reacted to accounting scandals by dismissing stock options as a way to compensate top executives. Our analysis suggests that the poor performance of stock option plans might be due to the lack of a good audit mechanism. Thus, a proper way to tackle this issue would be to improve the efficiency of the audit activity, instead of giving up stock options altogether, which looks like throwing away the baby with the water.

A better understanding of how the audit activity should be designed and regulated — including the interplay between internal and external auditors — seems to be an interesting line of future research. The insight we provide here points to the need of linking such analysis to the features of the compensation contract between top managers and shareholders.
References


